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Data Communication:

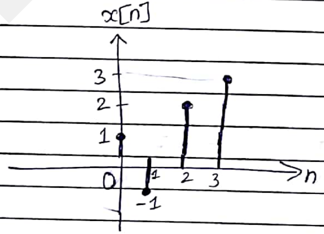
References:

1) "Data Communications and networking", Bebrouz A Forouzan
2) "Data and Computer Communications", William Stallings
3) "Signal Analysis", Rajeev prajapati

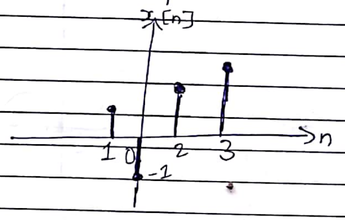
Chapter-1 (8 marks)

Signal.

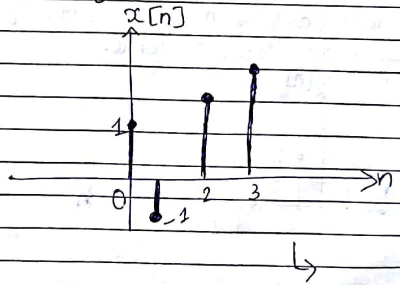
$x[n] = \{1, -1, 2, 3\}$



$x[n] = \{\overset{-1}{1}, \underset{\uparrow}{\overset{0}{-1}}, \overset{1}{2}, \overset{2}{3}\}$



$x[n] = \{\underset{\overset{\uparrow}{0}}{1}, -1, 2, 3\}$



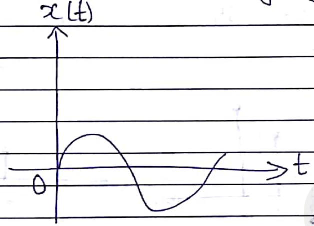
Signal:

Signal is a function of one or more independent variables which contains some useful informations. Signal can be classified as:

i) Analog Signal
ii) Discrete Signal

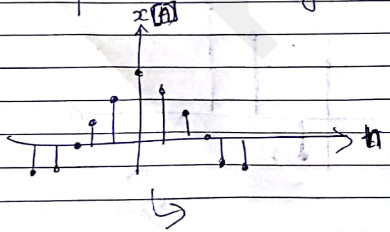
* Analog Signal:

Analog signal is that type of signals whose function can be defined at everytime instance. The independent variable 't' is enclosed in parenthesis( ) as $x(t)$ to represent analog signal.



* Discrete Signal:

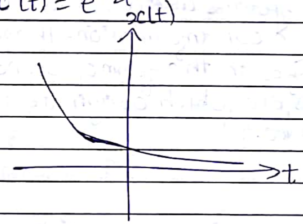
Discrete signals are those signals whose functions can be defined only at integer time instance. The independent variable 'n' is enclosed in [ ] as $x[n]$ to represent discrete signal.



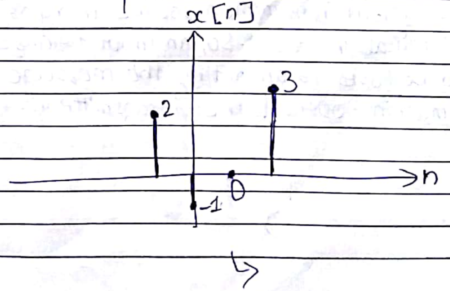
Communication System is the process for exchange of information between transmitter and receiver through channel. If the transmitting signal is analog signal, then the communication system is called Analog - Communication System. FM broadcasting is an example of it. On the other hand, if the transmitting signal is digital signal, then the Communication System is called Digital Communication System. Different Computer networkings are an example of digital Communication system.

Q. Draw the following signals:

a) $x(t) = e^{-2t}$



b) $x[n] = \{2, \underset{\uparrow}{-1}, 0, 3\}$



* interference: Change in amplitude or energy of Signal.
* distortion: Change in shape of signal.

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* Analog Communication System:

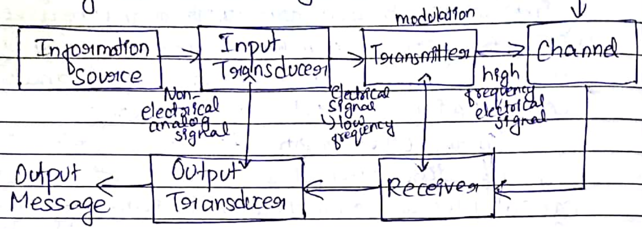

Fig: Block Diagram of Analog Communication System.

1. Information Source:
Communication Systems are designed to communicate a message or information. There can be various messages in the form of sound Signal, text, pictures, etc which originates in the information source.

2. Input Transducer:
A Transducer is a device which converts non-electrical signal into an electrical signal. The message from information source may or may not be electrical in nature. So, an input transducer is used before transmitting the message from information source to the transmitter.

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3. Transmitter:
The function of transmitter is to process the electrical signal obtained from different aspects. Modulation is the main function of the transmitter. Inside the transmitter, signal processing such as amplification and restriction of range of audio frequency are achieved.

4. Channel:
The function of channel is to provide a connection between transmitter and receiver. There are generally two types of channel:
i) Point-to-point channel (Wires, optical-fibre, etc).
ii) Broadcast Channel (Satellite Communication, TV broadcasting, etc).

Noise is an unwanted signal which tends to interfere with the required Signal. Noise is always random in nature and has greatest effect in signal at the channel.

5. Receiver:
The main function of receiver is to reproduce the message signal in electrical form from the distorted received signal. This reproduction of the original signal is accomplished by a process known as demodulation.

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6. Output Transducer:

Output transducer is the final stage which is used to convert an electrical message signal into its original form.

Finally, Information is transmitted from transmitter to the Receiver end.

Digital Communication System (DCS):-

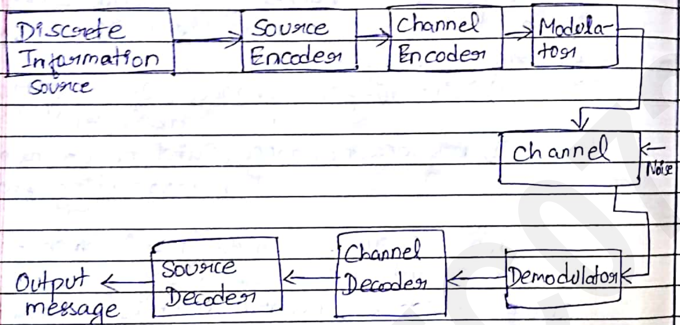


Fig: Block Diagram of Digital Communication System.

1) Discrete Information Source:

The information source produces a message signal which is not in the form of binary digits. The output of discrete information source consist of sequence of discrete symbols.

2) Source Encoder:

The symbols produced by the information source are given to the source encoder. This symbols cannot be transmitted directly so they are first converted into digital form (binary sequence of 1's and 0's). The group of these binary sequence is called Code word.

3) Channel Encoder:

Channel Encoder add some redundant (unnecessary) binary bits to the input sequence with some properly defined logic to avoid error in Communication.

4) Modulator:

Digital Modulators (ASK, FSK, PSK,) convert the input binary sequence of 1's and 0's to analog signal waveform.

5) Channel and Noise:

Communication between transmitter and receiver is established through communication channel. It maybe wirelines, wireless or fibre optic channels. Noise is unwanted signal which tends to interfere the signal in the channel.

6) Demodulator:

At the receiving end, the demodulator converts modulated signals into sequence of binary bits.

7) Channel Decoder:

Channel Decoder at the receiver is used to reconstruct the error-free bit sequence.

8) Source Decoder:

Source Decoder perform reverse operation as that of Source encoder i.e. it converts binary output to the symbol.

Hence, the transmitted symbols is received at the receiver.

* Advantages of digital Communication System (DCS):

i) DCS are cheaper compared to ACS because of advanced made in IC technology.
ii) Data encryption (Security) permits only allowed receiver to detect the message.
iii) In DCS, using multiplexing technique many data can be merged and transmitted over common channel.
iv) Large amount of noise can be tolerated.

* Disadvantages of DCS:

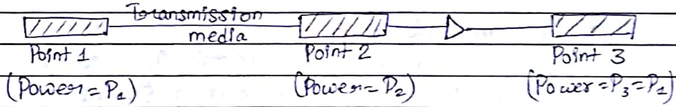
i) More transmission bandwidth is required for DCS.
ii) No. of A/D and D/A have to be used.

Transmission Impairments:

Signal travel through transmission media which are not perfect. This imperfection cause impairment (hazard) in the signal. This means the transmitted signals and the received signals are not same which is undesirable. There are three type of impairment and they are:

1. Attenuation
2. Distortion
3. Noise

1) Attenuation (loss of energy):



Attenuation means loss of energy when signal travels through medium. To commensate these attenuation or loss, amplifiers are used to amplify the signal. Decibel (dB) measures the relative strength of a signal at two different points. dB is +ve if a signal is amplify and -ve if the signal is attenuated. If P_1 and P_2 are powers of signal at points 1 and 2 then,

$$dB = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$$

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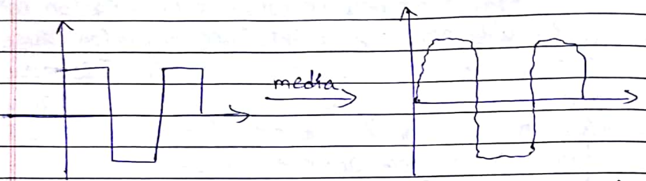
2) Distortion :



Transmitter signal   Receiver signal

Distortion means that the signal changes its form or shape. It is due to delay of signal arriving ~~of th~~ at the destination.

3) Noise:

Noise is an unwanted signal which causes problem in transmission of signal.
Some Noises are:-

* Thermal Noise:-
Due to random motion of electrons in a wire which creates an extra signal not originally sent by transmitter.

* Induced Noise:-
Comes from transmitter circuit itself.

* Cross talk:-
It is the effect of one wire on the other wire. One wire acts as sending antenna and other acts as receiving antenna.

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* Impulse Noise:-
Comes from power lines and lightening.

Chapter-2

Signals and systems.

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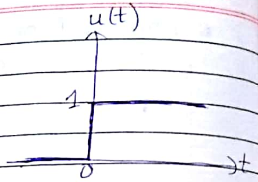
Basic Signals:

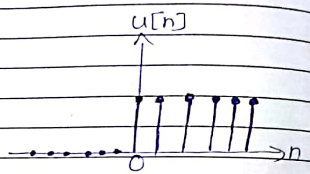
1) Unit step signal:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



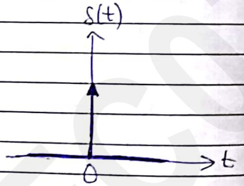
or,

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



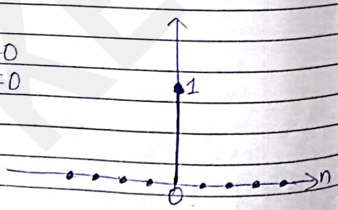
2) Delta function / Impulse function / Unit Impulse function:

$$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$



Or,

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$



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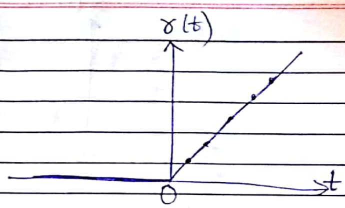
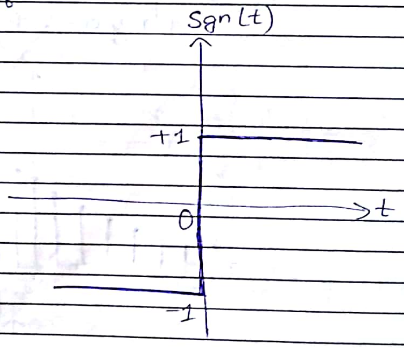
3) Ramp Signal:

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



4) Signum Signal:

$$Sgn(t) = \begin{cases} +1 & \text{for } t > 0 \\ 0 & \text{for } t = 0 \\ -1 & \text{for } t < 0 \end{cases}$$

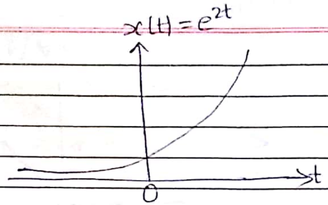


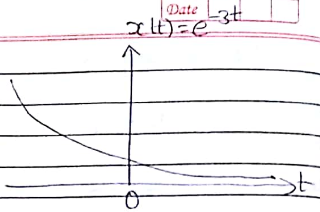
5) Exponential Signal:

a) Continuous time exponential signal, $x(t) = e^{at}$

Case I: If $a > 0$, then $x(t) = e^{at}$ is exponentially growing ~~grouping~~ signal.

Case II: If $a < 0$, then $x(t) = e^{at}$ is exponentially decaying signal.



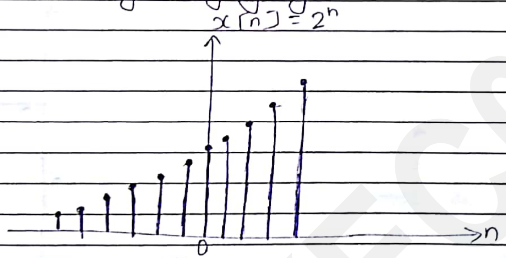


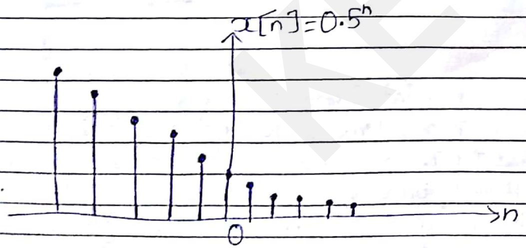
b) Discrete time Exponential signal :

$$x[n] = a^n$$

Case-I : If $a > 1$, then $x[n] = a^n$ is exponentially growing signal.

Case-II : If $0 < a < 1$, then $x[n] = a^n$ is exponentially decaying signal.



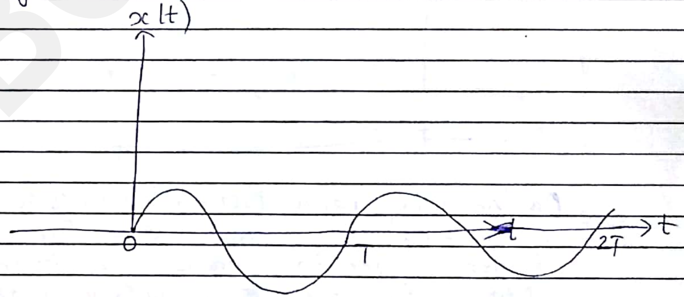


Classification of Signal :-

1) Periodic and Aperiodic Signal :

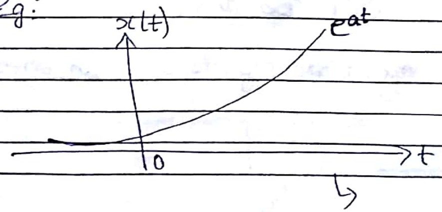
Periodic signal is that type of signal which has a definite pattern and repeats over and over with certain time period. The signal $x(t)$ is said to be periodic with period 'T' if $x(t+T) = x(t)$.

Eg:



On the other hand, a signal is said to be aperiodic if it doesnot repeat. The signal $x(t)$ is said to aperiodic with period 'T' if $x(t+T) \neq x(t)$.

Eg:

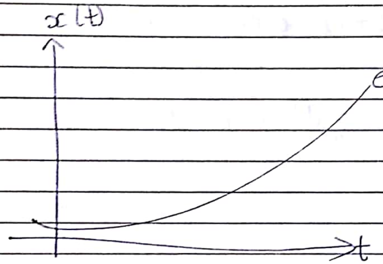


→ All periodic signals are power signals.
→ Most of aperiodic signals are energy signals.

2) Deterministic and Random Signal:

Those signals which have a regular pattern and can be characterized mathematically are called Deterministic Signal.
The nature and amplitude of deterministic signal at any time can be predicted.

Eg:



On the other hand, Non-deterministic signal is one which is of random nature. The pattern of such a signal is quite irregular.
Noise is an example of random signal.

3) Energy Signal and Power Signal:

Energy signal is one which has finite energy and zero average power. Hence, $x(t)$ is an energy signal if;

$$0 < E < \infty \text{ and } P = 0.$$

Most of aperiodic signals are an energy signals.

On the other hand, Power signal is one which has finite power and infinite energy. i.e.

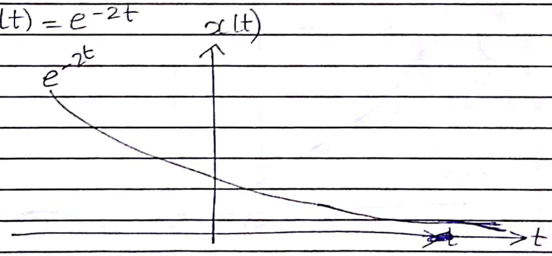
$$0 < P < \infty \text{ and } E = \infty.$$

All periodic signals are power signals.

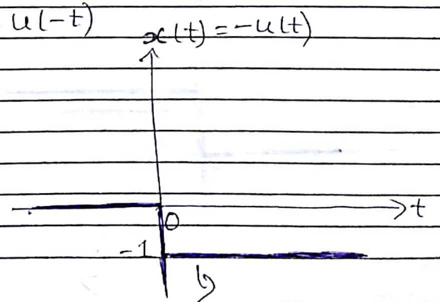
Those signals which doesn't satisfy above two conditions are called neither energy nor power signal. Ramp signal is an example of it.

Q.# Draw following signals:

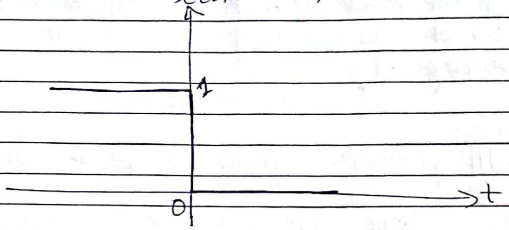
1) $x(t) = e^{-2t}$



2) $x(t) = u(-t)$

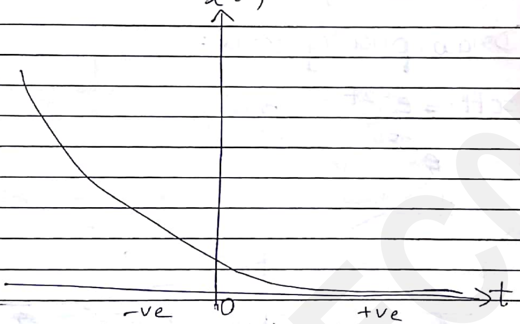


$x(t) = u(-t)$

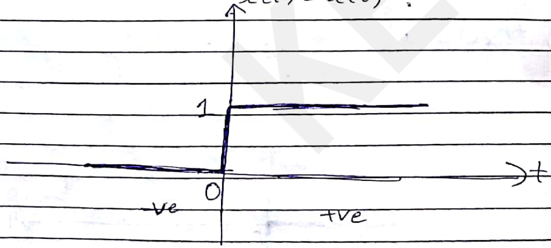


3) $x(t) = e^{-2t} \cdot u(t)$

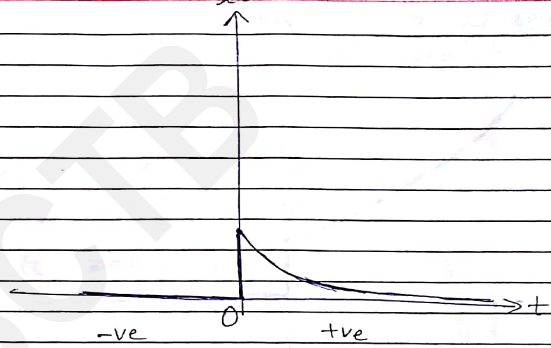
$x(t) = e^{-2t}$



$x(t) = u(t)$

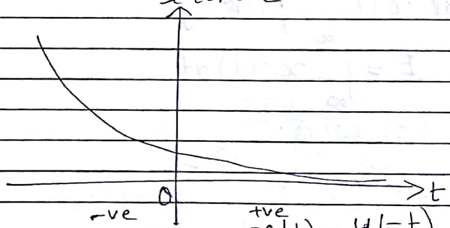


$x(t) = e^{-2t} \cdot u(t)$

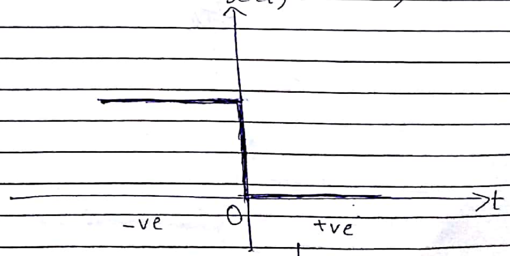


4) $x(t) = e^{-2t} u(-t)$

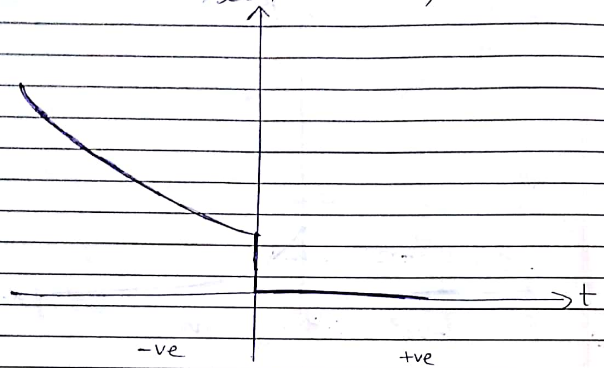
$x(t) = e^{-2t}$



$x(t) = u(-t)$



$x(t) = e^{-2t}.u(-t)$



* <u>Note:</u>

If $x(t)$ be any signal then, energy of signal $x(t)$ is given by;

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

and power is;

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

Q. Justify whether following signals are energy signal or power signal.

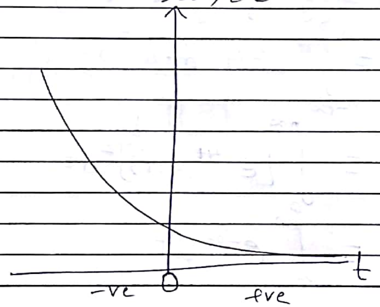
i) $x(t) = e^{-4t}.u(t)$

ii) $x(t) = \begin{cases} 1 & \text{for } -1 \le t \le 0 \\ e^{-2t} & \text{for } 0 \le t \le \infty \end{cases}$

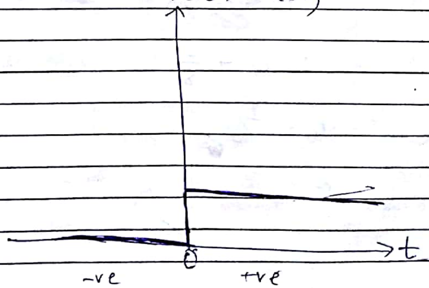
iii) $x(t) = e^{-a|t|}$ $(a > 0)$.

i)

$x(t) = e^{-4t}$



$x(t) = u(t)$



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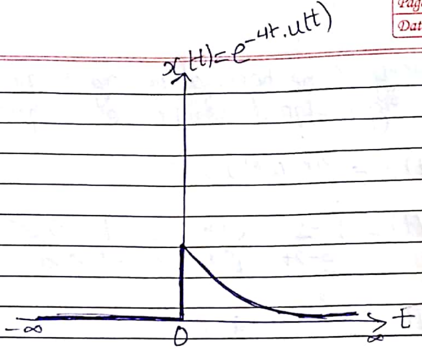

The energy of signal $x(t)$ is given by;

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{0}^{\infty} [e^{-4t}.\overset{1}{u(t)}]^2\,dt$$

$$= \int_{0}^{\infty} e^{-8t}\,dt$$

$$= \left[\frac{e^{-8t}}{-8}\right]_0^{\infty}$$

$$= 0 + \frac{1}{8}$$

$$\therefore E = 1/8$$

$e^{-2t} = e^{-2\cdot 0} = e^0 = 1$

$2\cdot e^{-2t} = 2\cdot e^{-2\cdot 0} = 2\cdot e^0 = 2$

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Again,

Power is given by;

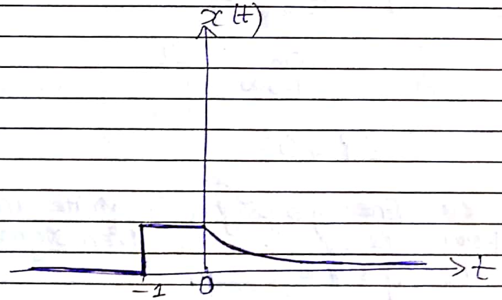
$$P = \lim_{T\to\infty} \frac{E}{T}$$

$$= \lim_{T\to\infty} \frac{(1/8)}{T}$$

$$\therefore P = 0$$

Since, Energy of given signal is finite and power is zero. So, the given signal is energy signal.

iii)



The energy of signal $x(t)$ is given by;

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-1}^{0} (1)^2\,dt + \int_{0}^{\infty} (e^{-2t})^2\,dt$$

$$= [t]_{-1}^{0} + \left[\frac{e^{-4t}}{-4}\right]_{0}^{\infty}$$

$$= [0+1] + 0 + \frac{1}{4}$$

$$= 1 + \frac{1}{4}$$

$$\therefore E = 5/4$$

Again,

Power is given by;

$$P = \lim_{T\to\infty} \frac{E}{T}$$

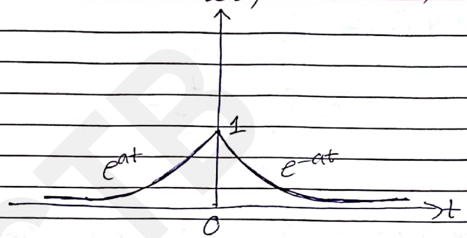
$$= \lim_{T\to\infty} \frac{5/4}{T}$$

$$\therefore P = 0$$

Since, Energy of given signal is finite and power is zero. So, the given signal is energy signal.

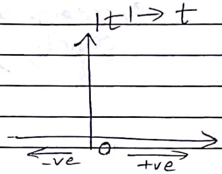
Pri 2)

$$x(t) = e^{-a|t|} \quad (a>0)$$



Hint:

$$|t| = \begin{cases} +t & \text{for } t>0 \\ -t & \text{for } t<0 \end{cases}$$



$$x(t) = \begin{cases} e^{-at} & \text{for } t>0 \\ e^{at} & \text{for } t<0 \end{cases}$$

The energy of signal is;

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-\infty}^{0} (e^{at})^2 dt + \int_{0}^{\infty} (e^{-at})^2 dt$$

$$= \left[\frac{e^{2at}}{2a}\right]_{-\infty}^{0} + \left[\frac{e^{-2at}}{-2a}\right]_{0}^{\infty}$$

$$= \frac{1}{2a} - 0 + 0 + \frac{1}{2a} = \frac{2}{2a} = \frac{1}{a}$$

$\therefore$ Power $(P) = \lim_{T \to \infty} \frac{E}{T}$

$= \lim_{T \to \infty} \frac{1/a}{T}$

$= 0$

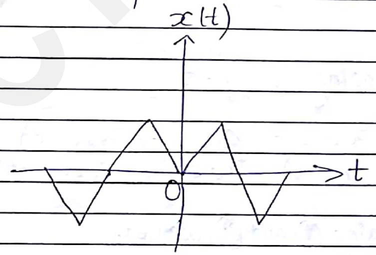
Since, Energy of given signal is finite and Power is zero. So, the given signal is energy signal.

Classification of Signal (Continue):

4) Even and odd signal:

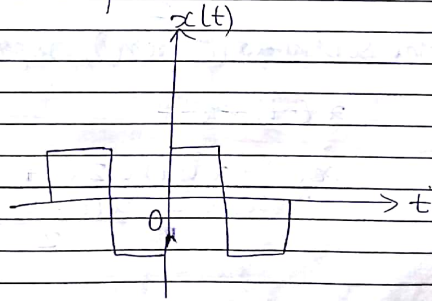
Any signal $x(t)$ is said to be an even signal if $x(t) = x(-t)$. That means, the signal is symmetric about y-axis.

For example:-



On the other hand, signal $x(t)$ is said to be an odd signal if $x(t) = -x(-t)$.

For example:-



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* Finding even and odd part of signal:

Any signal $x(t)$ is a combination of even and odd part of a signal i.e. ~~x~~

$$x(t) = x_e(t) + x_o(t) \quad \text{---(i)}$$

Where,

$x_e(t)$ = even part of signal

$x_o(t)$ = odd part of signal

Now,

Replacing t by $-t$,

$$x(-t) = x_e(-t) + x_o(-t)$$

or, $$x(-t) = x_e(t) - x_o(t) \quad \text{---(ii)}$$

Now, adding (i) and (ii), we get,

$$x(t) + x(-t) = 2x_e(t)$$

$$\therefore x_e(t) = \frac{x(t) + x(-t)}{2}$$

Again, Subtracting (ii) from (i), we get,

~~$x(-t) - x(t)$~~

$$x(t) - x(-t) = 2x_o(t)$$

$$\therefore x_o(t) = \frac{x(t) - x(-t)}{2}$$

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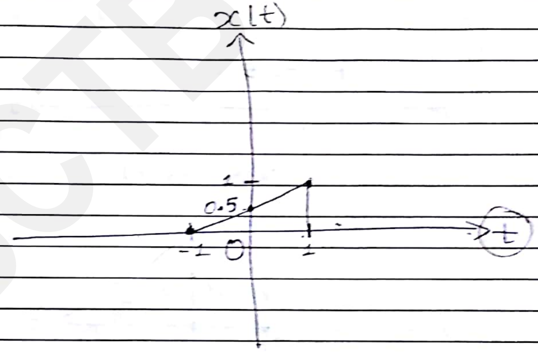
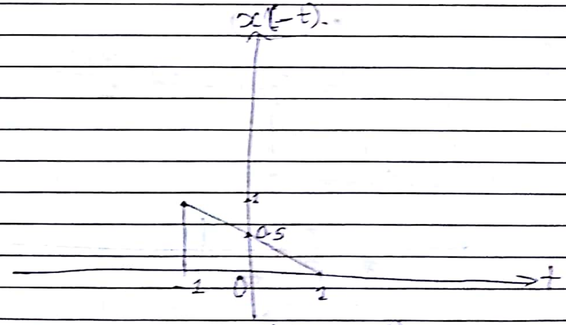
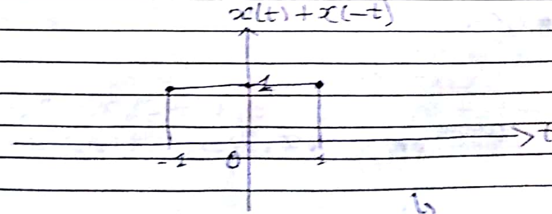
Q. Find even and odd part of signal:

$x(t) = 0.5(t+1)$ for $-1 \le t \le 1$.

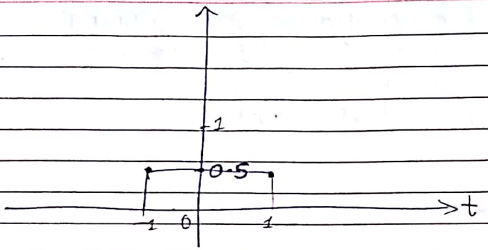
⇒



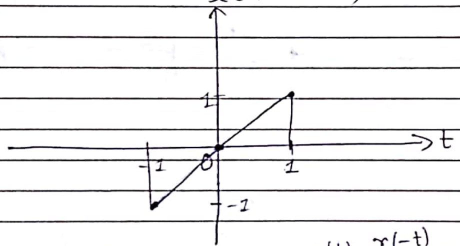




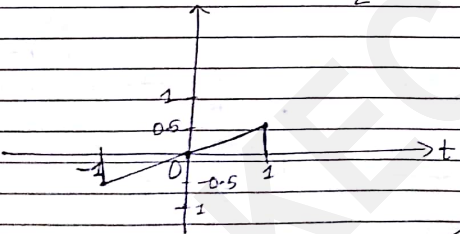
$$x_e(t) = \frac{x(t)+x(-t)}{2}$$



$x(t) - x(-t)$



$$x_o(t) = \frac{x(t)-x(-t)}{2}$$



★ Notations:
- Present output : $y(t)$
- Present input : $x(t)$
- Past inputs: $x(t-1), x(t-2), x(t-3), \ldots$
- ~~Past~~ future ~~o~~ inputs: $x(t+1), x(t+2), x(t+3), \ldots$

★ System and properties of System:

System may be defined as a set of elements or functional blocks which are connected together and produces an output in response to an input signal. The response or output of a system depends upon transfer function of the system. Various types of filters, amplifiers are an example of System.

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = f\{x(t)\}$$

Figure : System.

There are several Properties of System:

1. System with or without memory:-

Any System is said to have memory (dynamic System) if present output of a system depends upon past inputs. A system is said static or without memory if present output of a system depends upon present input only.

Eg of System with memory:

$$y(t) = x(t-1)$$
$$y(t) = x^2(t) + x(t-3)$$

Eg of system without memory:

$$y(t) = x^3(t)$$

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2. Causal and Non-Causal System:

If the present output of the system depends on ~~either~~ present inputs and/or past inputs, then the system is called Causal system. For example:

$$y(t) = x(t-1)$$
$$y(t) = x^2(t)$$
$$y(t) = x^3(t) + 2x(t-2)$$

On the other hand, Non-Causal system depends on future inputs. Since, Future inputs are practically not-realizable, all the practical systems are causal system.

For eg: $y(t) = x(t+3)$

~~3. Invertibility:~~

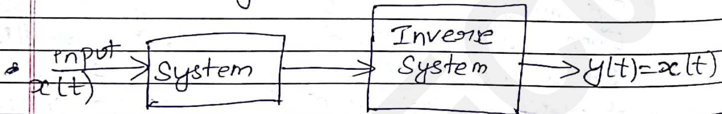

Figure: Invertibility.

A system is said to be invertible if the input of the system can be recovered from output of the system. All practical communication systems must be invertible.

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4. Linearity:

A system is said to be linear if it satisfy principle of superposition. If $x_1(t)$ and $x_2(t)$ be the inputs of a system producing an output $y_1(t)$ and $y_2(t)$ respectively, then, for linear system following equation must be satisfied:

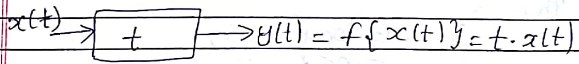
i.e. $f\{a.x_1(t) + b.x_2(t)\} = a\,y_1(t) + b\,y_2(t)$

Otherwise, a system is called non-linear system.

Q. Check whether following system are linear ~~or~~ ~~not~~ system or not?

i) $y(t) = t.x(t)$
ii) $y(t) = \log\{x(t)\}$

i ⇒ soln



We know that,

$y(t)$ is a function of $f\{x(t)\}$.

~~$y(t) = f\{x(t)\} = t.x(t)$~~

Then,

$y_1(t) = f\{x_1(t)\} = t.x_1(t)$

$y_2(t) = f\{x_2(t)\} = t.x_2(t)$

$$\therefore a y_1(t) + b y_2(t) = a.t x_1(t) + b.t x_2(t) \quad \text{---} (i)$$

Also,

$$f\{a x_1(t) + b x_2(t)\} = t.\{a x_1(t)\} + \{b x_2(t)\}$$

$$= \cancel{t\{a x_1(t) + b x_2(t)\}}$$

$$= a.t x_1(t) + b.t x_2(t) \quad \text{---} (ii)$$

Since, eqn (i) and (ii) are same. So, the given system is linear.

ii) Soln

We know that,

$y(t)$ is a function of $f\{x(t)\}$.

$$y(t) = f\{x(t)\}$$

$$= \log\{x(t)\}$$

Then,

$$y_1(t) = \log\{x_1(t)\}$$

$$y_2(t) = \log\{x_2(t)\}$$

$$\therefore a y_1(t) + b y_2(t) = a.\log\{x_1(t)\} + b.\log\{x_2(t)\} \quad \text{---} (i)$$

Also,

$$f\{a x_1(t) + b x_2(t)\} = \log\{a x_1(t) + b x_2(t)\} \quad \text{---} (ii)$$

Since, eqn (i) and (ii) are ~~same~~ not are same. So, the given system is not linear.

5. Time Invariance:

A system is said to be time invariant if a time shift of the input signal leads to an identical time shift in the output signal. If input of a system is $x(t)$, then output $y(t) = f\{x(t)\}$.

Now, if input is delayed by t_0 then, for time invariant system, following condition must be satisfied;

i.e. $f\{x(t-t_0)\} = y(t-t_0)$

Otherwise, the system is said to be time-variant system.

Q. Check whether a system $y(t) = t.x(t)$ is time invariant or variant?

Soln

We know that,

$y(t)$ is a function of $f\{x(t)\}$.

$$y(t) = f\{x(t)\}$$

$$= t.\{x(t)\}$$

Then,

$$y(t-t_0) = (t-t_0).x(t-t_0) \quad \text{---} (i)$$

Also,

$$f\{x(t-t_0)\} = t.x(t-t_0) \quad \text{---} (ii)$$

Since, eqn (i) and (ii) are not same. So, the given system is ~~not~~ time variant.

Q. $y(t) = x^2(t)$

=) We know that,

$y(t)$ is a function of $f\{x(t)\}$

$\cancel{y(t - t_0) = f\{x(t-t_0)\}}$

$$y(t) = f\{x(t)\}$$
$$= x^2(t)$$

$$y(t - t_0) = x^2(t - t_0) \quad \text{— (i)}$$

$$f\{x(t - t_0)\} = x^2(t - t_0) \quad \text{— (ii)}$$

Since, eqn (i) and (ii) are same. So, the given system is time invariant.//

6. Stability:

Any system is said to be BIBO (bounded input bounded output) if input and output of a system are bounded or finite. If $x(t)$ and $y(t)$ be the input and output of a system, then for BIBO stable system, then following conditions must be satisfied:

$$|x(t)| \le M_x < \infty$$
$$|y(t)| \le M_y < \infty$$

* Transformation of independent variables:

1) Time Shifting:

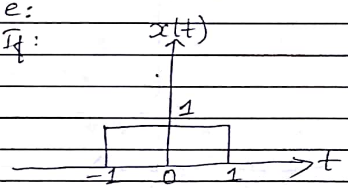
$$x(t) \xrightarrow{\text{time shifting}} x(t+a)$$

Case I: If $a < 0$, then $x(t+a)$ is delayed version of $x(t)$ by 'a' unit.

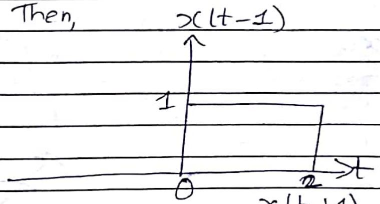
Case II: If $a > 0$, then ~~x(t)~~ $x(t+a)$ is ~~delayed~~ advanced version of $x(t)$ by 'a' unit.

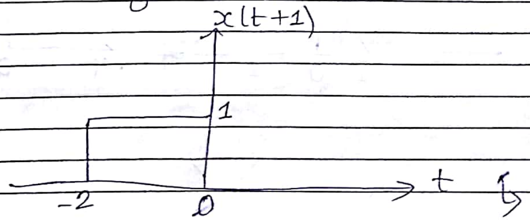
Example:

If:



Then,





2) Time Scaling:

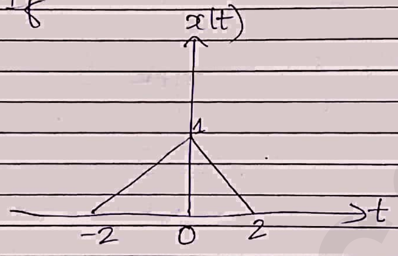
$x(t) \xrightarrow{\text{time scaling}} x(at)$

Case I: If $a>1$, then $x(at)$ is compressed version of $x(t)$.

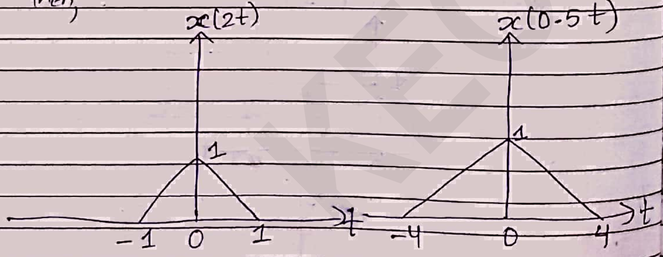
Case II: If $a<1$, then $x(at)$ is expanded version of $x(t)$.

Example:

If



Then,

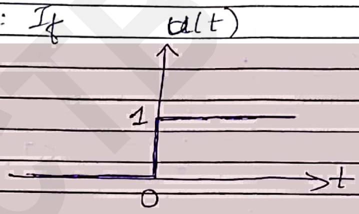


first shift and then scaling the shifted signal

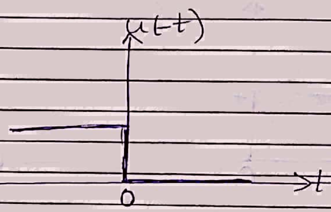
3) Time Inversion:

$x(t) \longrightarrow x(-t)$

Eg: If



Then,



* Precedence Rule:

If we have to perform time shifting and time scaling operation on a same signal, then always perform time shifting before time scaling or inversion.

$x(t) \longrightarrow x(at+b)$

$x(t) \xrightarrow{\text{shifting}} x(t+b) \xrightarrow{\text{scaling}} x(at+b)$

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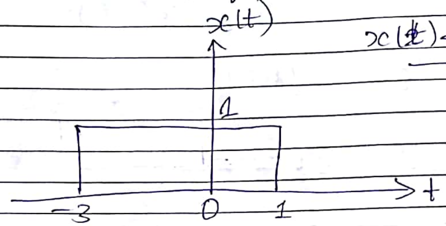
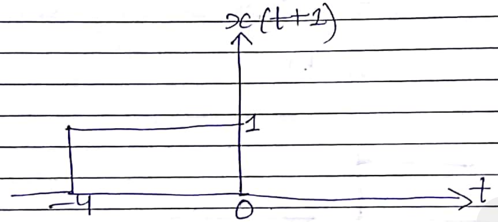
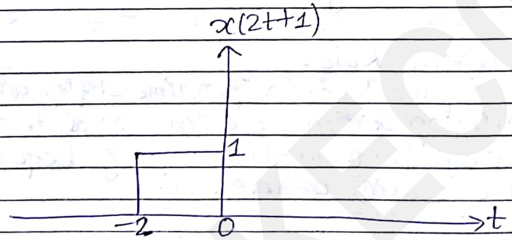
Q. Draw $x(2t+1)$ from given $x(t)$.

$x(t) \rightarrow x(t+1) \rightarrow x(2t+1)$



⇒





Chapter-3.

Signal Analysis.

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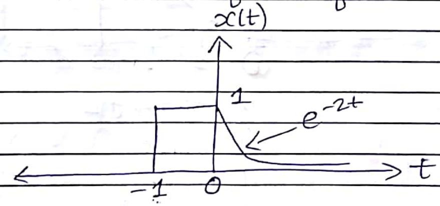
* **Linear Time Invariant (LTI) system:**

Any system which satisfy both linearity and time invariance property is known as LTI system. Impulse response denoted by '$h(t)$' is output of system when input is particularly delta or impulse signal, i.e. $h(t) = f\{\delta(t)\}$

Now, the output of LTI system is obtained by convolution operation between input signal $x(t)$ and impulse response $h(t)$ as

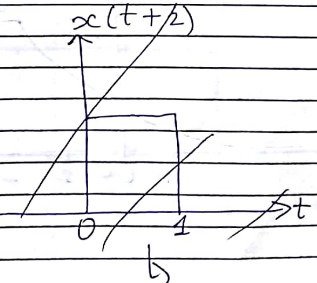
$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

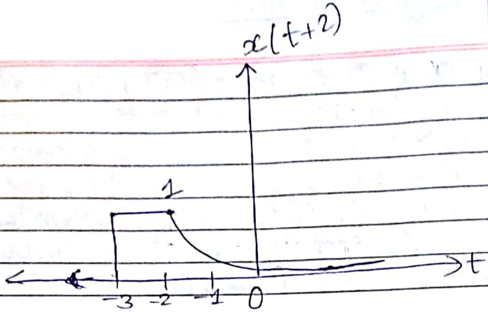
Q. Draw $x(2-t)+2$ from following signal:

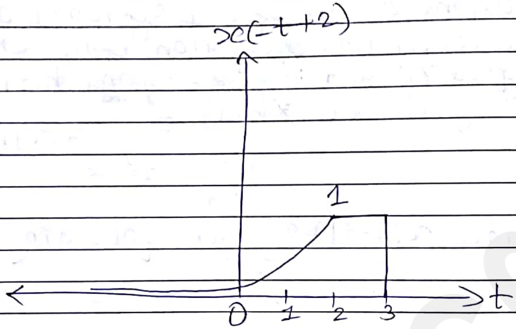


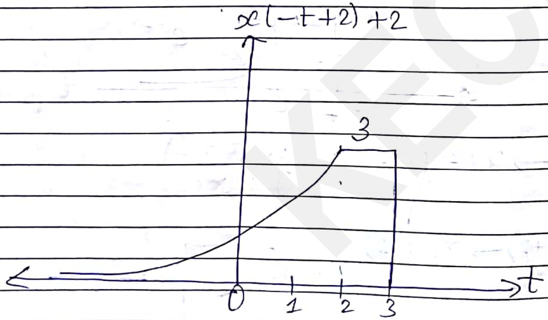
$x(t) \rightarrow x(2-t)+2$

$x(t+2) \rightarrow x(-t+2)$

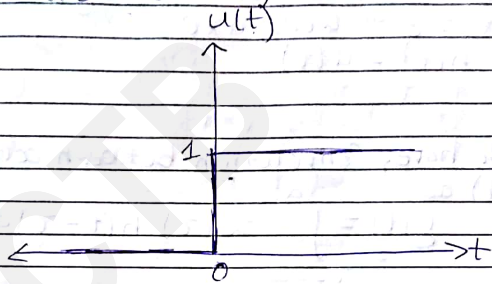


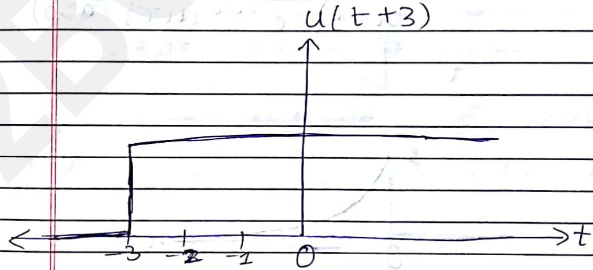


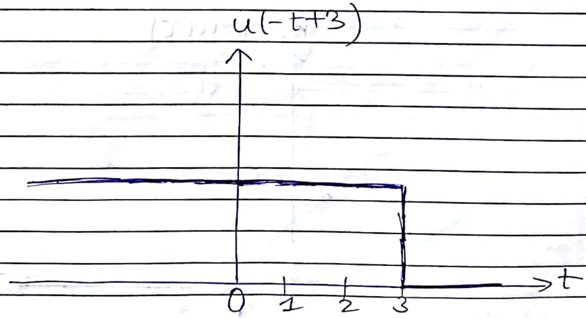




Q. Draw u(-t+3).







Q. Find convolution between two signals

$x(t) = e^{-at}.u(t) \; (a>0)$

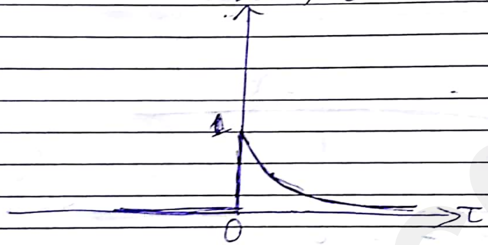
and, $h(t) = u(t)$

⇒ soln

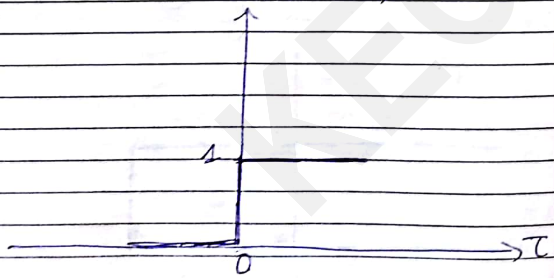
We have, convolution between $x(t)$ and $h(t)$ as:

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(\tau) = e^{-a\tau} u(\tau) \; (a>0)$



$h(\tau) = u(\tau)$



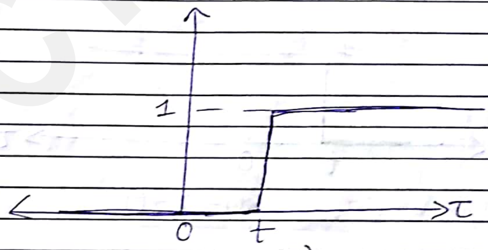
↳

$h(\tau) \longrightarrow h(-\tau+t)$

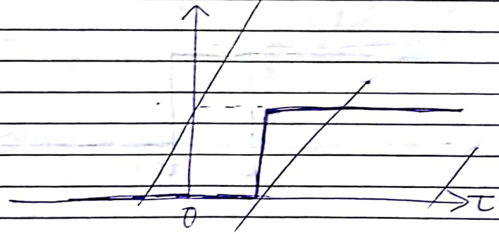
$h(\tau) \searrow h(\tau+t) \nearrow h(-\tau+t)$

case-I) If $t<0$,

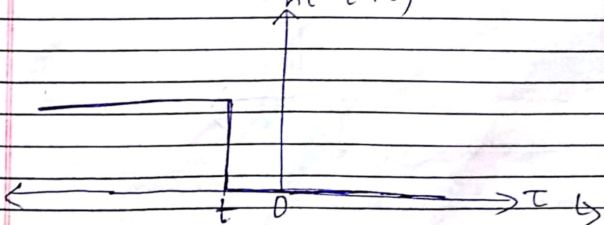
$h(\tau+t)$



$h(-\tau+t)$



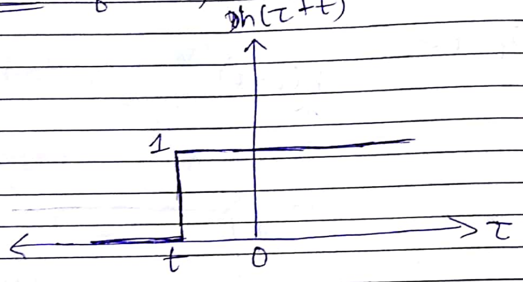
$h(-\tau+t)$

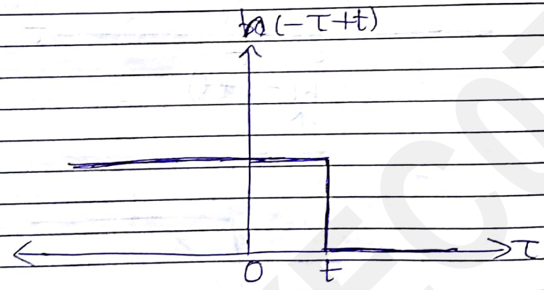


↳

In this case, there is no overlapping between $x(t)$ and $h(t-\tau)$. So, $y(t) = 0$ for $t<0$.

Case-II : If $t>0$,





In this case, there is overlapping between $x(t)$ and $h(t-\tau)$ from 0 to t. So,

$$y(t) = \int_0^{t} [e^{-a\tau}\cancel{u(\tau)}]\cdot[\cancel{u(t-\tau)}]\,d\tau$$

$$= \int_0^t e^{-a\tau}\,d\tau$$

$$= \left[\frac{e^{-a\tau}}{-a}\right]_0^t$$

$$= -\frac{1}{a}\left[e^{-a\tau}\right]_0^t$$

$$= -\frac{1}{a}\left[e^{-at} - e^{-a\cdot 0}\right]$$

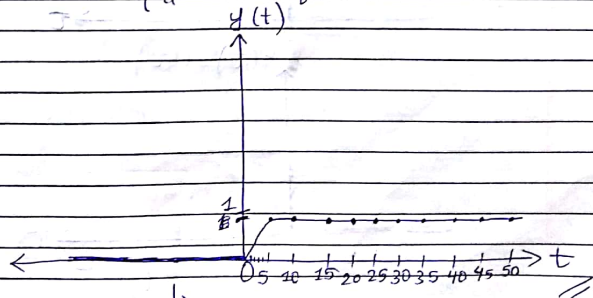
$$= -\frac{1}{a}\left[e^{-at} - 1\right]$$

$$\cancel{= \frac{e^{-at}}{a}}$$

$$= \frac{1}{a}\left(1 - e^{-at}\right)$$

So,

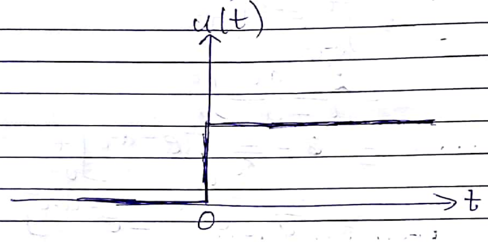
$$y(t) = \begin{cases} 0 & \text{for } t<0 \\ \frac{1}{a}(1-e^{-at}) & \text{for } t>0 \end{cases}$$



Q. Find convolution between two unit-step signals and comment on the result.

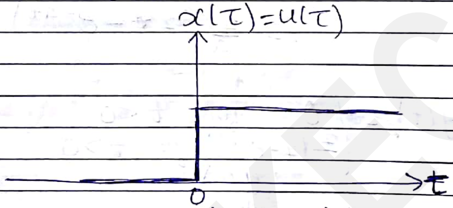
⇒ $x(t) = u(t)$ $\quad u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$

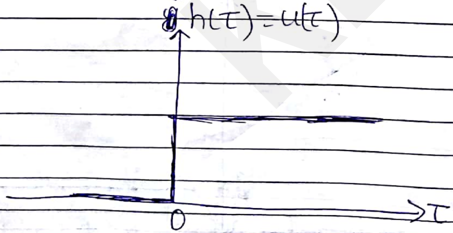
$h(t) = u(t)$



We have, Convolution between $x(t)$ and $h(t)$ as:

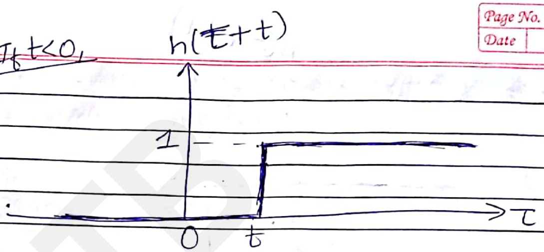
$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

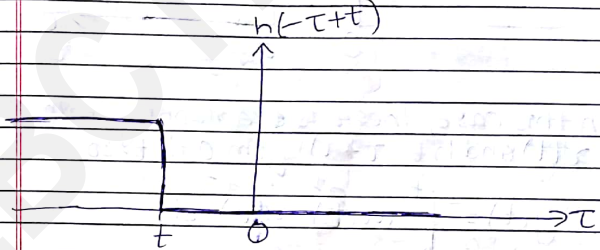




↳

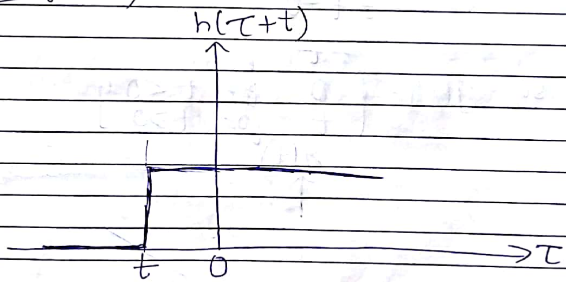
Case-I: If $t<0$,





In this case, there is no overlapping betn $x(t)$ and $h(t-\tau)$, so, $y(t) = 0$ for $t<0$

Case-II: If $t>0$,



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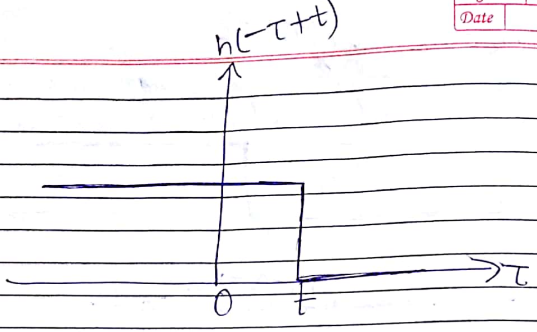


In this case, there is overlapping betn $x(t)$ and $h(-\tau+t)$ from 0 to t. So,

$$y(t) = \int_0^t (\cancel{u(\tau)}^{1} \cdot \cancel{u(\tau)}^{1})\, d\tau$$

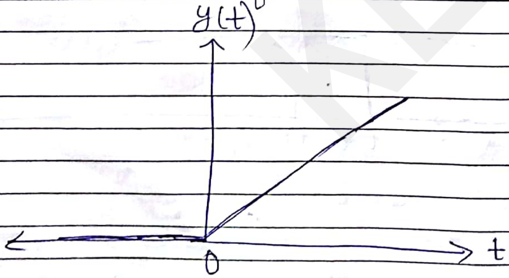
$$= \int_0^t d\tau$$

$$= [\tau]_0^t$$

$$= t - 0$$

$$= t$$

So, $y(t) = \begin{cases} 0 & \text{for } t<0 \\ t & \text{for } t>0 \end{cases}$



→ Consulation of two unit step-signals result in ramp signal.

Q. Find Convolution between following two signals:

$$x(t) = e^{-at}.u(t) \quad (a>0)$$

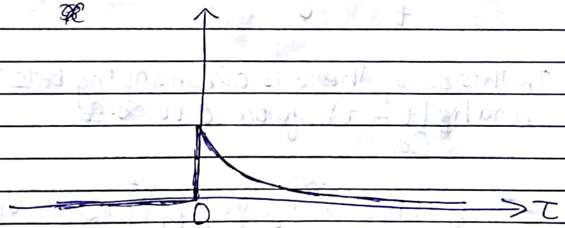
and, $h(t) = e^{at}.u(-t) \quad (a>0)$

⇒ Soln

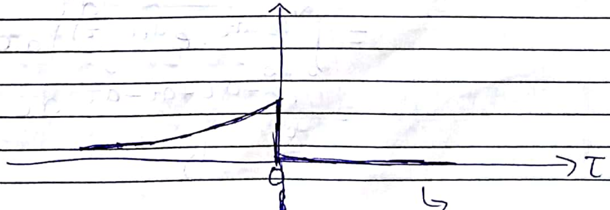
We have, Convolution between $x(t)$ and $h(t)$ as:

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

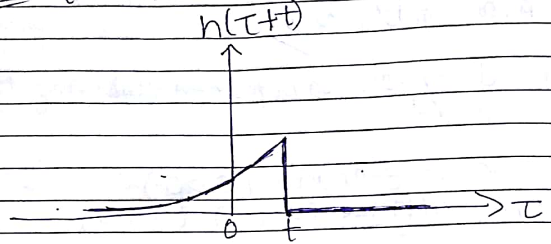
$$x(\tau) = e^{-a\tau}.u(\tau) \quad (a>0)$$

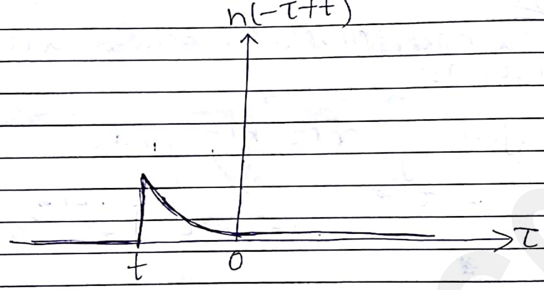


$$h(\tau) = e^{a\tau}.u(-\tau)$$



Case-I: If $t<0$,





In this case, there is overlapping betn $x(t)$ and $h(t-\tau)$ from 0 to ∞. So,

$$y(t) = \int_0^{\infty} \left[e^{-a\tau}\cancelto{1}{u(\tau)}\right]\left[e^{a(t-\tau)}\cancelto{1}{u(-(t-\tau))}\right] d\tau$$

$$= \int_0^{\infty} \left(e^{-a\tau} . e^{a(t-\tau)}\right) d\tau$$

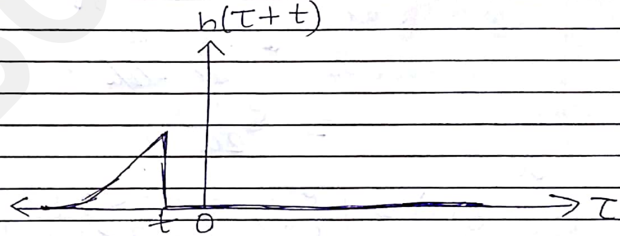
$$= \int_0^{\infty} e^{-a\tau+at-a\tau}\, d\tau$$

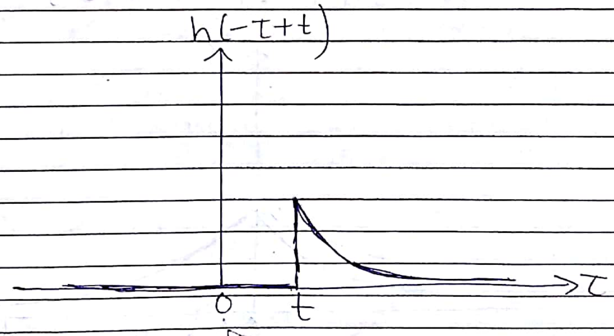
$$= \left[\frac{e^{-2a\tau+at}}{-2a}\right]_0^{\infty}$$

$$= 0 + \frac{e^{at}}{2a}$$

$$= \frac{e^{at}}{2a}$$

Case-II: If $t>0$,





In this case, there is overlapping betn $x(t)$ and $h(t-\tau)$ from t to ∞.

So,

$$y(t)=\int_t^{\infty}\left(e^{-a\tau}\cancel{u(\tau)}\right)\left[e^{a(t-\tau)}\cancel{u}(t-\tau)\right]d\tau$$

$$=\int_t^{\infty} e^{-2a\tau+at}\,d\tau$$

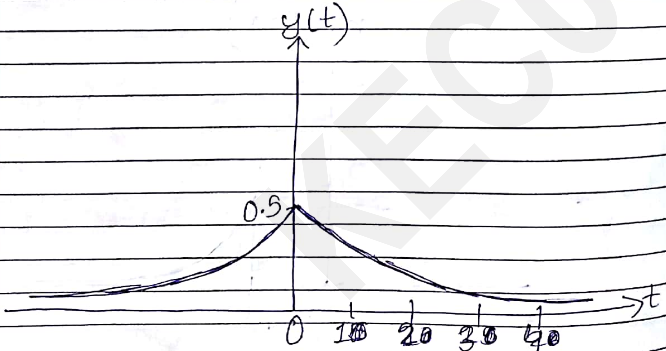
$$=\left[\frac{e^{-2a\tau+at}}{-2a}\right]_t^{\infty}$$

$$=\cancel{\frac{e^{at}}{-2a}}\;\cancel{\frac{e^{at}}{-2a}}$$

$$=\frac{e^{-at}}{2a}$$

So,

$$y(t)=\begin{cases} e^{at}/2a & \text{for } t<0 \\ e^{-at}/2a & \text{for } t>0 \end{cases}$$



— F.S gives only periodic signal frequency response but cannot of aperiodic signal.

— F.T → for both periodic and aperiodic signal.

* Fourier Series Representation of Continuous time Signal:

If $x(t)$ be any periodic signal, then, Fourier Series equations are:

$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{— synthesis eqn}$$

Where, a_k is called fourier series-coefficient and is given by;

$$a_k=\frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\,dt \quad \text{— Analysis eqn.}$$

Q. For the signal, $x(t)=\sin\omega_0 t$, find its fourier series coefficients.

* Note: If 1, sin, cos, $1+\sin 2\omega_0 t+\cos(2\omega_0 t+\pi/4)$ or combinations of it questions appear in $x(t)$, then use synthesis equation.

⇒ Soln

We have, Synthesis eqn as:

$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Or, $\sin\omega_0 t=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

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$$\text{or, } \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\text{or, } \frac{e^{j\omega_0 t}}{2j} - \frac{e^{-j\omega_0 t}}{2j} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Now, Comparing the coefficients, we get,

$$a_1 = \frac{1}{2j}$$

$$a_{-1} = -\frac{1}{2j}$$

$$a_k = 0 \text{ for } k \neq \pm 1$$

Q. Find fourier Series coefficients of signal:

$$x(t) = 1 + \sin\omega_0 t + \cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$$

⇒ soln

We have synthesis eqn as:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\text{or, } 1 + \sin\omega_0 t + \cos\omega_0 t + \cos(2\omega_0 t + \pi/4) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\text{or, } 1 + \frac{e^{j\omega_0 t}}{2j} - \frac{e^{-j\omega_0 t}}{2j} + \frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2} + \frac{e^{j(2\omega_0 t + \pi/4)}}{2} + \frac{e^{-j(2\omega_0 t + \pi/4)}}{2}$$

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$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\text{or, } 1 + \frac{e^{j\omega_0 t}}{2j} - \frac{e^{-j\omega_0 t}}{2j} + \frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2} + \frac{e^{j2\omega_0 t}}{2} \cdot e^{j\pi/4} + \frac{e^{-j2\omega_0 t}}{2} \cdot e^{-j\pi/4} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\text{or, } 1 + \frac{e^{j\omega_0 t}}{2j} - \frac{e^{-j\omega_0 t}}{2j} + \frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2} + \frac{e^{j2\omega_0 t}}{2} \cdot e^{j\pi/4} + \frac{e^{-j2\omega_0 t}}{2} \cdot e^{j\pi/4} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\text{or, } 1 + e^{j\omega_0 t}\left(\frac{1}{2j} + \frac{1}{2}\right) - e^{-j\omega_0 t}\left(\frac{1}{2j} - \frac{1}{2}\right) + \frac{e^{j2\omega_0 t}}{2} \cdot e^{j\pi/4} + \frac{e^{-j2\omega_0 t}}{2} \cdot e^{-j\pi/4} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Comparing the Coefficients, we get,

$$a_0 = 1 \qquad a_{-1} = \frac{1}{2} - \frac{1}{2j}$$

$$a_1 = \frac{1}{2j} + \frac{1}{2}$$

$$a_2 = \frac{e^{j\pi/4}}{2} \qquad a_{-2} = \frac{e^{-j\pi/4}}{2}$$

$e^{-10}-e \over 2j$

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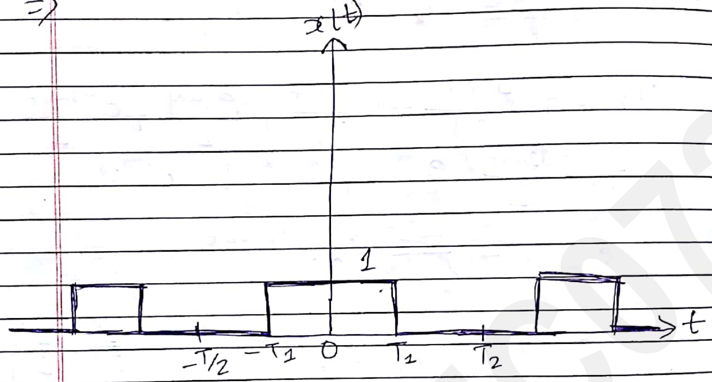
Q. Find Fourier Series Coefficients of following Signal and plot magnitude and phase spectrum for $T = 4T_1$.

$$x(t) = \begin{cases} 1 & \text{for } |t| < T_1 \longrightarrow -T_1 < t < T_1 \\ 0 & \text{for } T_1 < |t| < T/2 \longrightarrow -T_1 < t < -T/2 \\ & \qquad \qquad \qquad \hookrightarrow T_1 < t < T/2 \end{cases}$$

$\Rightarrow$



We have,
Fourier Series Coefficients as;

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_{-T_1}^{T_1} 1 \cdot e^{-jk\omega_0 t}\, dt$$

$\hookrightarrow$

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$$= \frac{1}{T}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-T_1}^{T_1}$$

$$= \frac{1}{T}\left(\frac{e^{-jk\omega_0 T_1} - e^{jk\omega_0 T_1}}{-jk\omega_0}\right)$$

$$\cancel{= \frac{1}{-jk\omega_0 T} \cdot \frac{2j}{\sin k\omega_0 T_1}}$$

$$\cancel{= -\frac{2}{k\omega_0 T} \sin k\omega_0 T_1}$$

$$= \frac{2\sin k\omega_0 T_1}{k\omega_0 T}$$

$$= \frac{2\sin k\omega_0 T/4}{k\omega_0 T}$$

$$= \frac{2\sin(k\pi/2)}{2k\pi}$$

$$\therefore a_k = \frac{\sin(k\pi/2)}{k\pi} \quad \text{for } k \neq 0$$

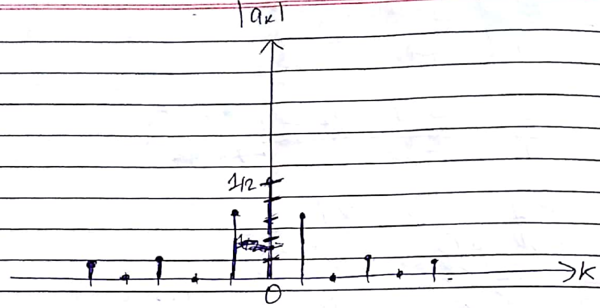
Using L-Hospital's Rule for $k=0$,

$$a_0 = \frac{\pi/2 \cos(\cancel{k}^{\,0}\pi/2)}{\pi}$$

$$= \frac{1}{2}$$

$$\therefore a_k = \begin{cases} 1/2 & \text{for } k = 0 \\ \dfrac{\sin(k\pi/2)}{k\pi} & \text{for } k \neq 0 \end{cases}$$

$\hookrightarrow$



Magnitude plot.

→ Since, there is no imaginary part. So, there is no phase plot. Since, $\tan^{-1}\left(\frac{Im^{0}}{Re}\right) = \tan^{-1} 0 = 0$

Fourier Transform of Continuous time signal:

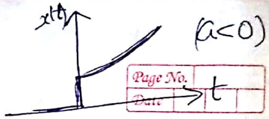
Fourier transform of any signal $x(t)$ is:

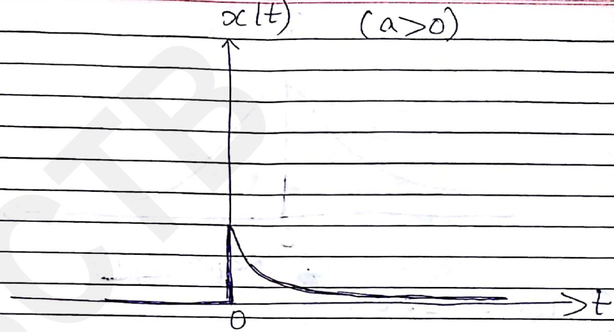
$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad \text{(Fourier transform eqn)}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)\, e^{j\omega t}\, d\omega \quad \text{[Inverse Fourier transform eqn]}$$

Q. Find Fourier transform of $x(t) = e^{-at}.u(t)$ $(a>0)$ and plot magnitude and phase spectrum

↳ =)





We have, Fourier transform eqn as;

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} \left[e^{-at}.\cancel{u(t)}^{\,1}\right] e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

$$= \frac{e^{0}}{a+j\omega}$$

$$= \frac{1}{a+j\omega}$$

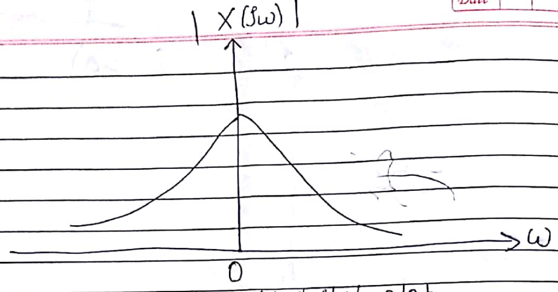
$$\therefore |x(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \left[|a+jb| = \sqrt{a^2+b^2}\right]$$

& $\angle x(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$

↳

$a+jb = \tan^{-1}(b/a)$

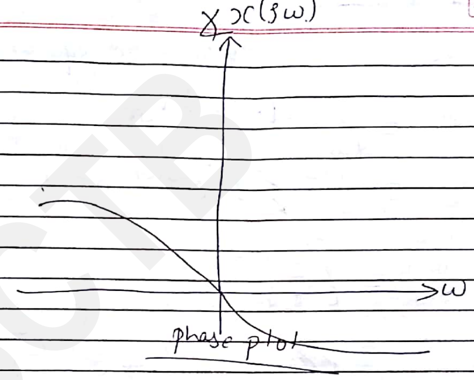
$\frac{1}{a+jb} = -\tan^{-1}(b/a)$



Magnitude plot.

Let, $a = 1 > 0$.

ω	$\lvert X(j\omega)\rvert = \frac{1}{\sqrt{1+\omega^2}}$	$\measuredangle X(j\omega) = -\tan^{-1}(\omega)$
25	0.0399	~~87.7093~~ -1.530817
20	0.0499	-1.520837
15	0.0665	-1.504228
10	0.0995	-1.471127
5	0.1961	-1.3734
0	1	0
-5	0.1961	1.3734
-10	0.0995	1.471127
-15	0.0665	1.504228
-20	0.0499	1.520837
-25	0.0399	1.530812



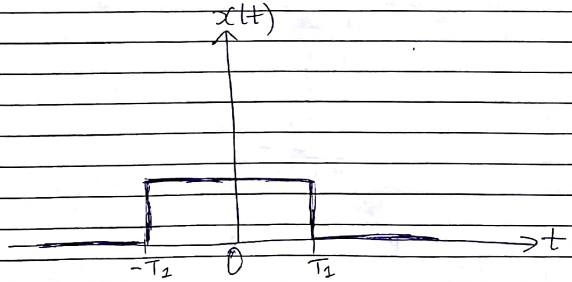
phase plot

* Magnitude plot is always even signal
* Phase plot is always odd signal.

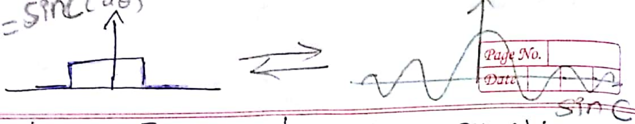
Q. Determine Fourier transform of signal

$$x(t) = \begin{cases} 1 & \text{for } -T_1 < t < T_1 \\ 0 & \text{Otherwise} \end{cases}$$

$\Rightarrow$



$\frac{\sin(a\theta)}{a\theta} = \mathrm{sinc}(a\theta)$



We have, Fourier transform eqn as;

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T_1}^{T_1} 1\, e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T_1}^{T_1}$$

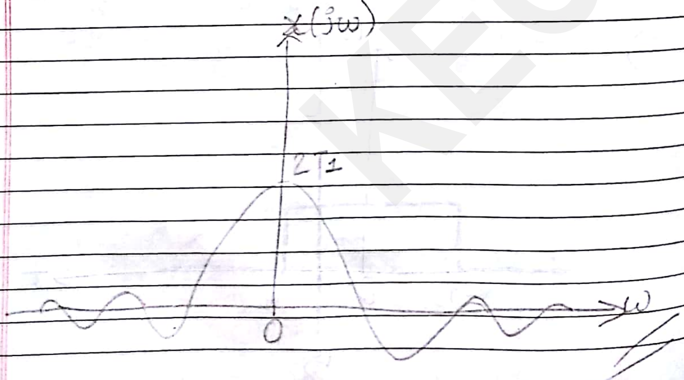
$$= \left[\frac{e^{-j\omega T_1}}{-j\omega} + \frac{e^{j\omega T_1}}{j\omega}\right]$$

$$= \cancel{\frac{2\sin\omega T_1}{j\omega}} \quad 2j \cdot \frac{\sin\omega T_1}{j\omega}$$

$$= \frac{2}{\omega} \cdot \sin\omega T_1$$

$$= 2 \cdot \frac{\sin(\omega T_1)}{\omega T_1} \times T_1$$

$$\therefore X(j\omega) = 2T_1\, \mathrm{sinc}(\omega T_1)$$



Chapter-5.

Data Encoding and Modulation.

* Modulation:

Modulation may be defined as a process by which some characteristics of carrier signal is ~~valid~~ varied according to message signal. The signal containing information to be transmitted is called Message signal. The carrier frequency is greater than the message frequency and the signal obtained from modulation process is known as modulated signal.

There are two types of modulation:

1) Analog Modulation:

If the carrier signal is continuous in nature, then the modulation process is called Analog Modulation. Amplitude Modulation (AM), frequency Modulation (FM) and phase modulation (PM) are an example of it.

2) Pulse Modulation:

If the carrier waveform is a pulse-type waveform, then the modulation process is called pulse-Modulation. Pulse-Amplitude Modulation (PAM), Pulse-Width Modulation (PWM), Pulse-code Modulation (PCM) are an example of Pulse Modulation.

* Need / Benefits of Modulation:-

i) Practicality of antenna size:

When freespace is used as a channel, messages are transmitted and received with the help of antenna. For efficient radiation and reception,

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the transmitting and receiving antenna size must be $\lambda/4$. If modulation is not used, then the length of antenna would be very high (impossible to construct) and if modulation is used, then it will be practical to design the antenna size.

ii) To remove interference:

These is always need to transmit multiple no. of signals over a same channel. If we transmit all these signals without modulation, then due to almost same frequency range of information signal, they get overlapped. The solution to this problem is to shift frequency one than another in frequency axis which is modulation.

iii) Reduction of Noise:

Noise cannot be eliminated completely but with the help of several modulation schemes, the effect of noise can be minimized.

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Amplitude Modulation (AM):

It is defined as a process in which amplitude of carrier wave $C(t)$ is varied according to message signal $m(t)$. ~~The~~

The standard form of AM wave is:

$$S(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

or,

$$S(t) = [A_c + m(t)] \cos 2\pi f_c t$$

and, modulation index of A.M is given by:

$$\mu = \frac{A_m}{A_c}$$

If $P_c = \frac{A_c^2}{2}$ is carrier power then total Power of AM is:

$$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

and efficiency of AM is:

$$\eta = \frac{\mu^2}{2+\mu^2} \times 100\%$$

Now,
We have 3 different cases depending upon value of μ.

Case-I: ~~If~~

Case I: If $\mu < 1$,

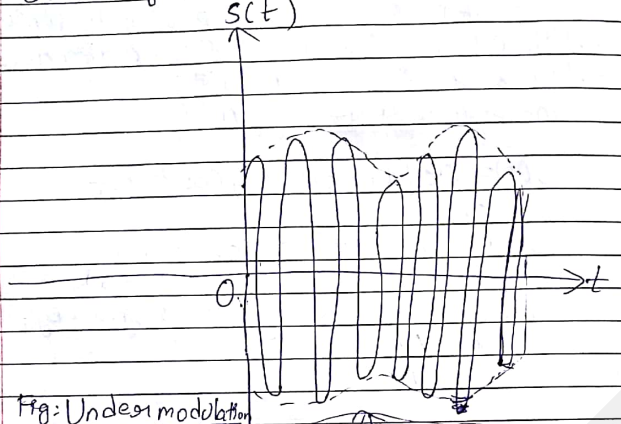
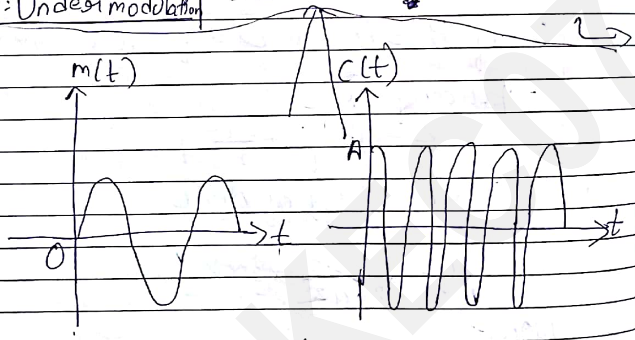
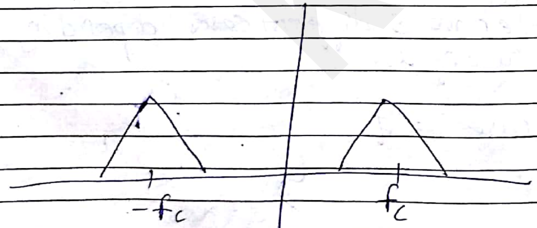


Fig: Undermodulation





Case II: If $\mu = 1$.

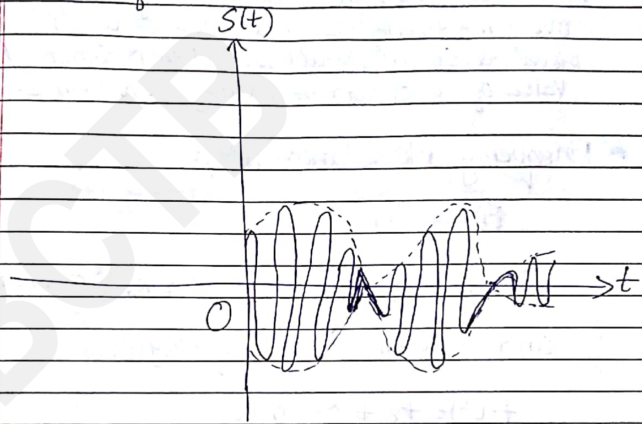


Fig: 100% modulation.

Case III: If $\mu > 1$.

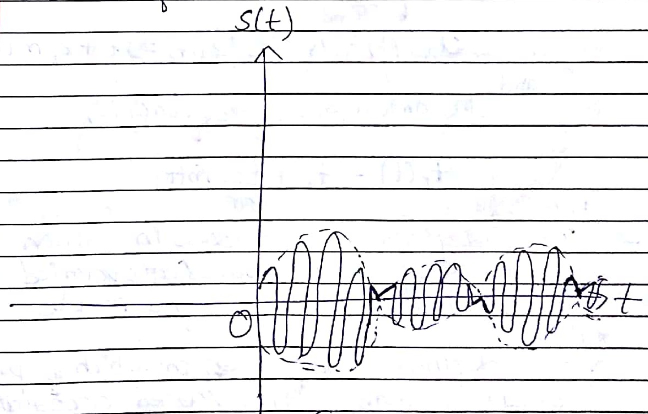


Fig: Over-modulation.

We cannot reconstruct message signal from over-modulated signal because of phase reversal. So, in practical communication, the value of μ should be always less than 1.

Frequency Modulation (FM):

FM eqn is;

$$u_{FM}(t) = A_c \cos\left[2\pi f_c + k_d \int_0^t m(t)\,dt\right]$$

and instantaneous frequency is;

$$f_i(t) = f_c + \frac{k_d}{2\pi} m(t)$$

Phase Modulation (PM):

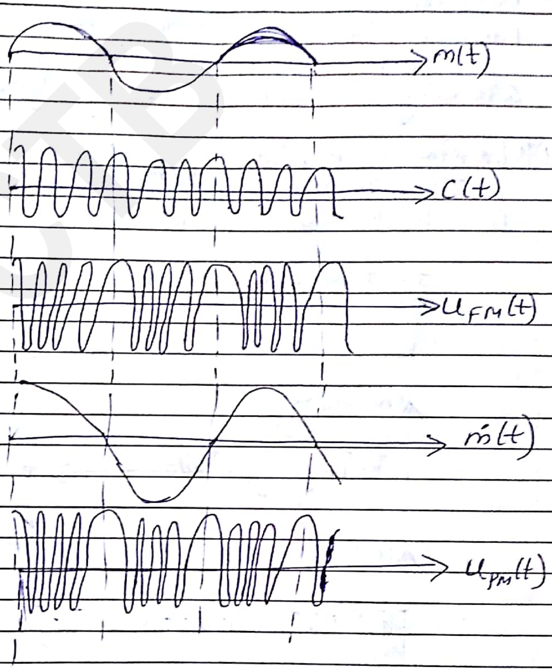
PM eqn is;

$$u_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

and, instantaneous frequency is,

$$f_i(t) = f_c + \frac{k_p}{2\pi} \dot{m}(t)$$

* FM:
→ It is defined as a process in which frequency of carrier wave c(t) is varied according to message signal m(t).
* PM:
→ It is defined as a process in which phase of carrier wave c(t) is varied according to derivative of message signal $\dot{m}(t)$.



The bandwidth of FM is given by Carlson's rule which is :

$BW = 2(\beta+1) f_m$ where, β is modulation index of FM and f_m is message frequency.

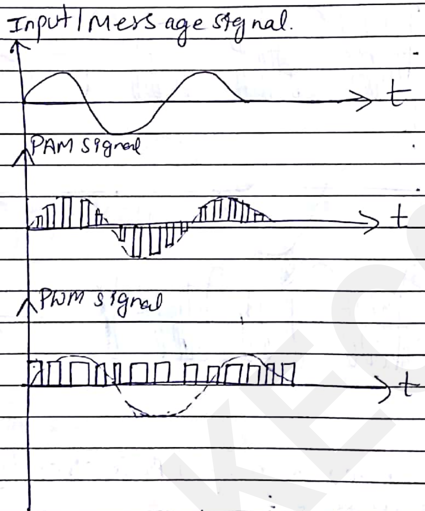
* Pulse ~~A~~ Modulation:

1) Pulse Amplitude Modulation (PAM):-

In PAM, amplitude of pulse is varied according to input or message signal.

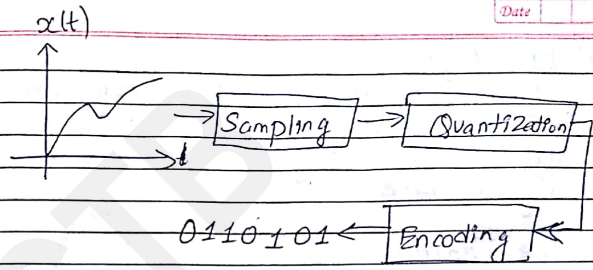
2) Pulse ~~Am~~ Width Modulation (PWM):-

In PWM, the pulse width is varied according to input or message signal.



3) Pulse Code Modulation (PCM):-

PCM is known as a process to encode analog signal into digital data



PCM is known as digital pulse modulation technique. It has 3 processes:

i) Sampling:

The first step in PCM is sampling. The analog signal is sampled every T_s second where T_s is sampling interval.
The inverse of sampling interval is sampling rate. i.e. $f_s = \frac{1}{T_s}$. The sampling rate is restricted to a certain value which is given by Nyquist Criteria i.e. $f_s \geq 2f_m$.

ii) Quantization:

The result of sampling is series of pulse with different amplitude values of the signal. These values cannot be used directly in the encoding process. So, Quantization is performed before encoding which converts the amplitude of signal into corresponding integer values.

iii) Encoding:

Finally, encoding converts the quantized inputs into corresponding bit streams and hence, the analog input signal is converted into digital data.

* Line Coding:

- Line Coding
  - Unipolar
  - NRZ (Non-Return to zero)
    - NRZ-L (Non-Return to zero-level)
    - NRZ-I (Non-Return to zero Inverted)
  - RZ (Return to zero)
  - Biphase
    - Manchester
    - Differential Manchester
  - Bipolar (AMI)

1) Unipolar:
- ~~One~~ 1 is encoded by positive value.
- 0 is encoded by zero voltage.

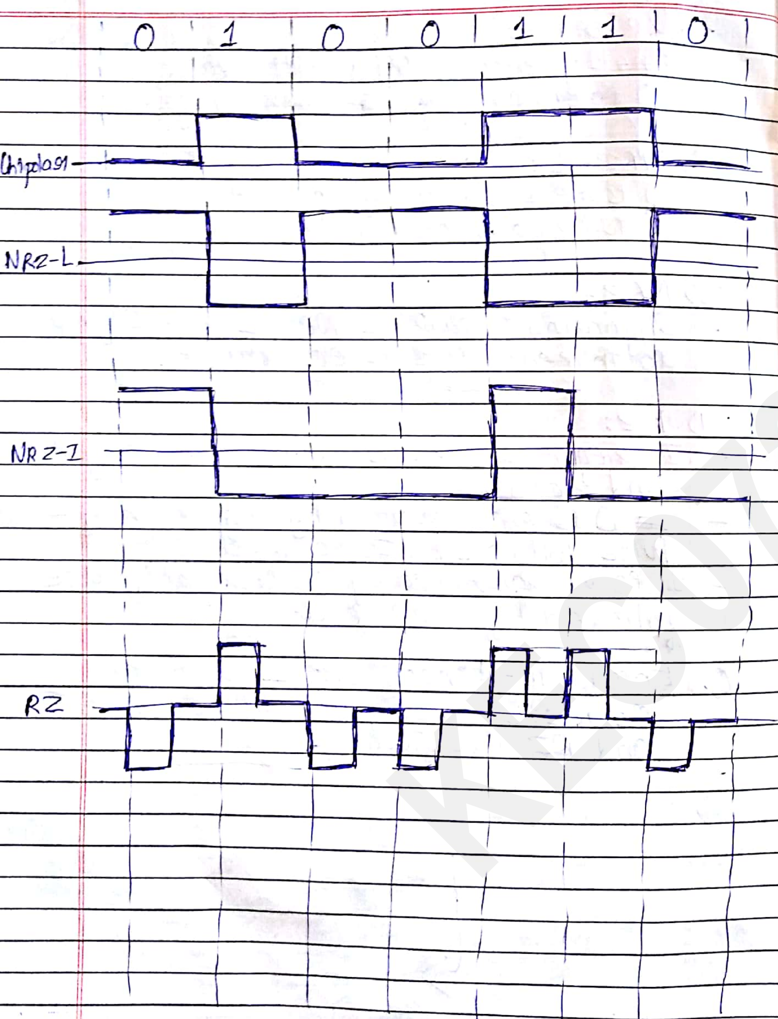
2) NRZ-L:
- 0 is represented by high-level.
- 1 is represented by low-level.

3) NRZ-I:
- Signaling is same as NRZ-L but signal is inverted if 1 is encountered.

4) RZ:
- It requires 3 values i.e. positive, negative and zero level.
- ~~Z~~ 0 is represented by negative half-pulse followed by zero level.
- 1 is represented by positive half pulse followed by zero level.

Q. Encode the binary ~~stream~~ stream 0100110 in unipolar, NRZ-L, NRZ-I and RZ line coding.

=>



5) Manchester encoding:

- 0 is represented by transition from high to low in mid of time ~~of tim~~ interval.
- 1 is represented by transition from low to high in mid of time interval.

6) Differential Manchester encoding:

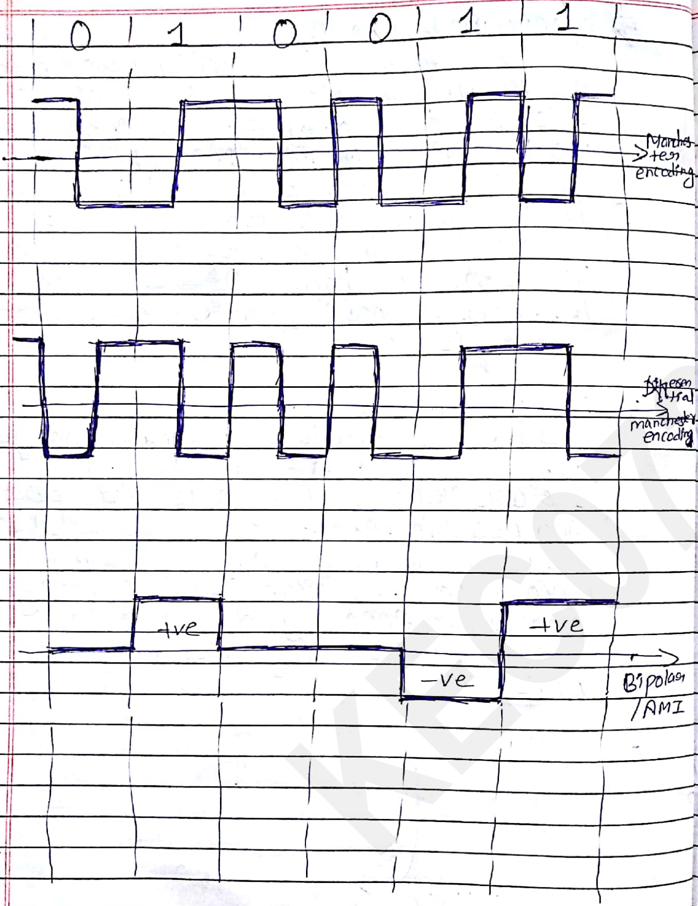
- It requires two level of signal changes to represent binary 0 ~~zero~~ but only one level of signal change to represent binary 1.

7) Bipolar (AMI):

- 0 level is used to represent binary ~~zero~~ 0.
- 1's are represent by alternate positive and negative values.

Q) Encode the bitstream 010011 using Manchester encoding, differential manchester encoding and Bipolar or AMI.

=)



* Digital Modulation:

Digital Modulation is the process of converting binary sequence into an analog signal. Modulation is terms as Shift keying in case of digital Modulation and hence we have 3 digital Modulation techniques and they are:-

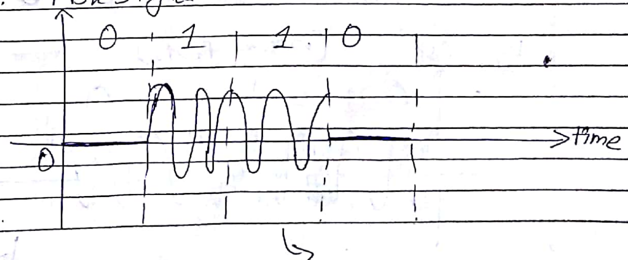
1) Amplitude Shift keying (ASK):
2) Frequency Shift keying (FSK).
3) Phase Shift keying (PSK)

1) ASK:

In ASK system, binary symbol 1 is represented by transmitting a sinusoidal carrier of fixed amplitude and fixed frequency where as, binary symbol 0 is represented by no carrier transmission. Mathematically,

$$s(t) = \begin{cases} A_c \cos 2\pi f_c t & \text{for sign symbol 1} \\ 0 & \text{for symbol 0} \end{cases}$$

Eg: ASK Signal



The ASK generation circuit is shown below:-

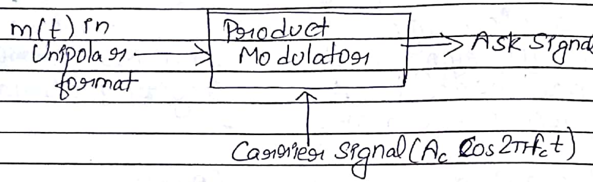


Fig: ASK generation Circuit.

For generation of ASK, message signal must be in Unipolar format. The message signal and carrier wave are fed to the product modulator producing ASK signal.

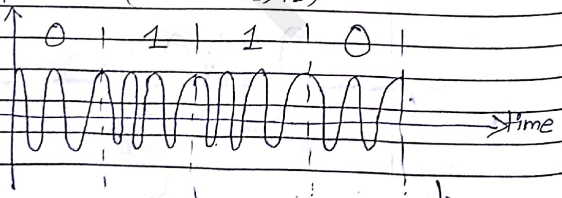
2) FSK:

In FSK system, two sinusoidal waves of same amplitude but different frequencies are used to represent binary 1 and 0.

Mathematically,

$$s(t) = \begin{cases} A_c \cos 2\pi f_1 t & \text{for symbol } 1 \\ A_c \cos 2\pi f_2 t & \text{for symbol } 0 \end{cases}$$

Eg: FSK (Assume: $f_1 > f_2$)



FSK generation circuit is shown below:-

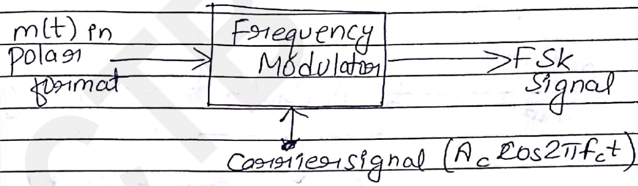


Fig: FSK generation Circuit.

[Polar: 0 → negative pulse of same height.
1 → positive pulse of same height.]

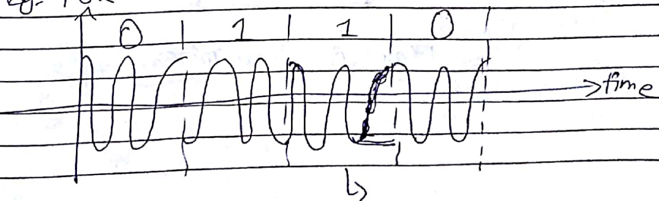
3) Phase Shift keying (PSK):

In PSK system, sinusoidal carrier wave of fixed amplitude and fixed frequency is used to represent both binary 1 and 0 except that carrier phase for each symbols differs by 180°.

Mathematically,

$$s(t) = \begin{cases} A_c \cos 2\pi f_c t & \text{for symbol } 1 \\ A_c \cos(2\pi f_c t + \pi) & \text{for symbol } 0 \end{cases}$$

Eg: PSK



PSK signal can be generated using following circuit:

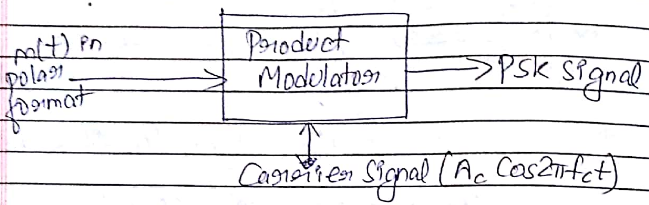


Fig: PSK generation circuit.

* Quadrature Amplitude Modulation (QAM):

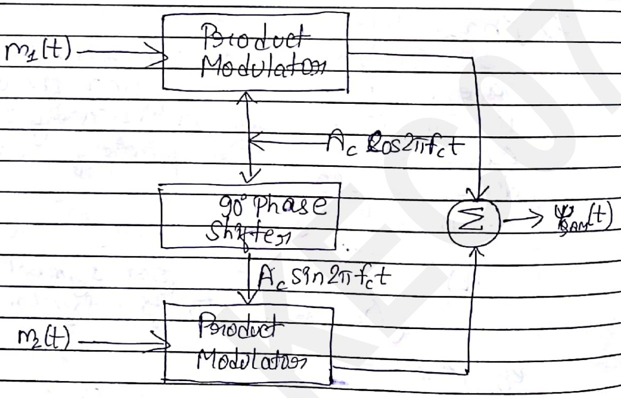


Fig: QAM transmitter.

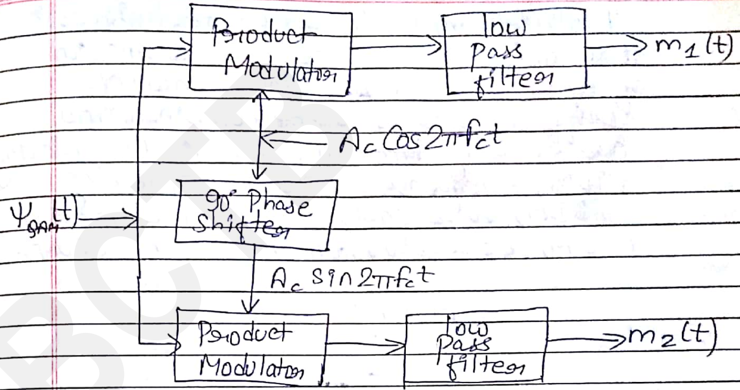


Fig: QAM Receiver.

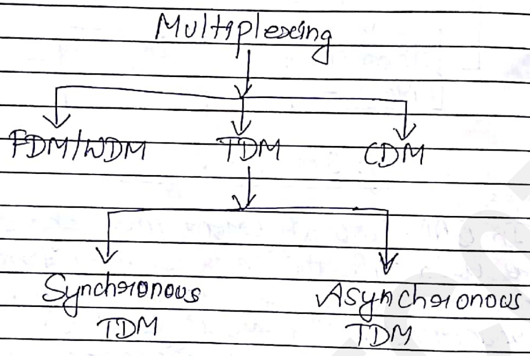
In QAM, two different message signals are used and the carrier for transmitting these message signals are always in phase-Quadrature (90° Phase difference) with each other. If two message signals to be transmitted are $m_1(t)$ and $m_2(t)$, then, Corresponding QAM signal is:

$$\psi_{QAM}(t) = m_1(t) \cdot A_c \cos 2\pi f_c t + m_2(t) \cdot A_c \cos 2\pi f_c t$$

Now, at the receiver side, $\psi_{QAM}(t)$ is applied to two product modulators followed by low pass filter. The output from upper branch is message $m_1(t)$ and output from lower branch is message $m_2(t)$.

Multiplexing and Spreading.

→ Multiplexing is the set of techniques that allows simultaneous transmission of multiple signal in a single channel. Different signals can be combined into one by using ~~the~~ Time Division Multiplexing (TDM), Frequency Division Multiplexing (FDM), Wavelength Division Multiplexing (WDM), Code Division Multiplexing (CDM).



* Applications of Multiplexing are:

i) Proper Utilization of channel can be done.
ii) It helps to transmit no. of message signals into a common channel.
iii) It reduces cost of transmission.
iv) Helps to develop efficient system.

* Frequency Division Multiplexing (FDM):

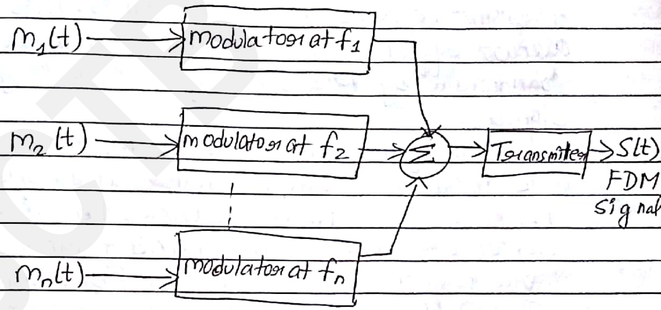


Figure: FDM transmitter.

BPF → Band Pass Filter

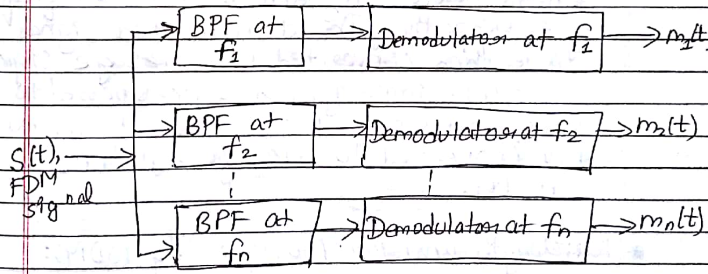


Figure: FDM Receiver.

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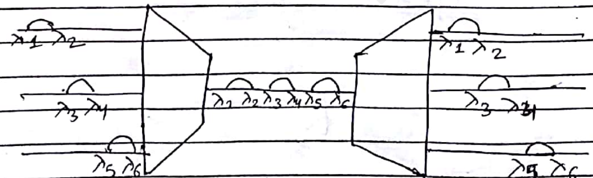
In FDM, available frequency band is divided into no. of frequency slots for transmitting no. of message signals. This allows efficient utilization of channel bandwidth by transmitting no. of message signals.

'n' no. of message signals i.e. $m_1, m_2, \ldots m_n$ are modulated using modulator at frequencies $f_1, f_2, \ldots f_n$ respectively. The modulated signals are then combined and transmitted via transmitter producing FDM signal. The FDM signal is then received at the receiver.

Now, the FDM signal is passed to Band Pass Filter at f_1 and then the signal is demodulated using demodulator at frequency f_1 respectively. The demodulated signals are then converted into message signals $m_1(t)$. Similarly, FDM signal is passed to other Band Pass Filter at $f_2, \ldots f_n$ and then finally converted to message signals $m_2(t), \ldots m_n(t)$.

* Wavelength Division Multiplexing (WDM):



Synchronous TDM: Easy, Inefficient
Asynchronous TDM: Efficient, Complex

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-> WDM is conceptually same as FDM.
-> Multiplexing and Demultiplexing involves light signals transmitted through fibre optic channel.
-> Narrow beam of light from different sources are combined to make a white band of light with high intensity signals.

* Time Division Multiplexing (TDM):

-> TDM is used when data capacity of the transmitting medium is greater than the requirement of each sending and receiving device.
-> Different signals are multiplexed into same channel in different time slots.
-> There are two types of TDM and they are:- Synchronous TDM and Asynchronous TDM.

i) Synchronous TDM:

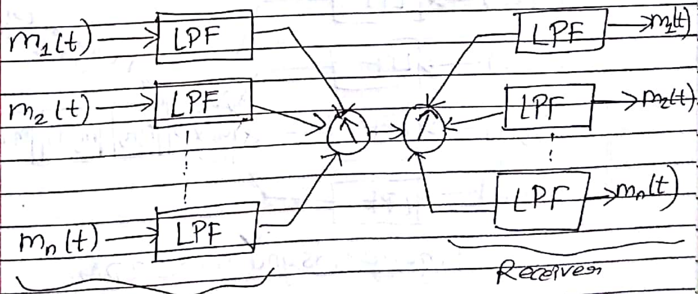

Fig: TDM Block.

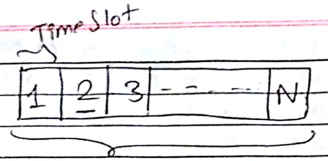

Figure: Time slot arrangement for TDM

-> In Synchronous TDM, the Multiplexer allocates exactly the same time slots to each device at all the time.

-> The time slot is unused when the device doesnot tramit the message or data which is its main disadvantage leading to waste of capacity.

-> However, Synchronous TDM is simple and easy to design and implement.

ii) Asynchoronous TDM:

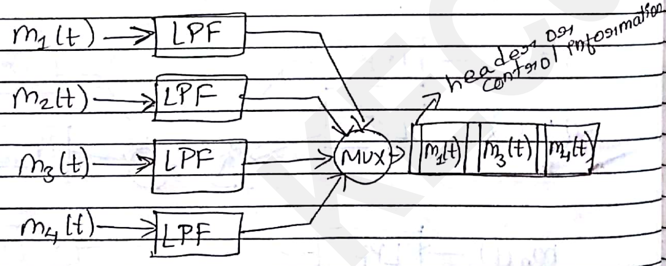

Figure: Asynchoronous TDM.

-> Asynchoronous TDM dynamically allocates the time slots on demand and hence time-slots are allocated depending upon the need only. This avoids or prevents waste of time slots. This method add the overhead or header or control information to the multiplexer and demultiplexer which increases system complexity. So, Asynchoronous TDM is an efficient system and complex also.

Chapter-8. $I \propto \frac{1}{P}$

Information Theory and Coding.

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* Information:

Consider a system which needs to transmit weather in city 'A' to a receiving station in City 'B'. Let us suppose that the weather as being sunny, cloudy, raining & foggy.

Each weather indication represents a source symbol and a sequence of weather information is a message. The information can be defined as amount of uncertainity that the receiver have about what is being sent. For instance, suppose that the receiver has following probability.

Weather Symbol	Probability
Sunny	0.65
Cloudy	0.20
Rainy	0.10
Foggy	0.05

Since, City A is nearly always sunny, being told that it is sunny conveys little information. On the other hand, if it is foggy, i.e. very surprising and conveys more information.

* Self information & Average Information (Entropy):

We define an information source (S) as a collection of discrete symbols (S_i) representing an event that the source transmit i.e.

$$S = \{S_1, S_2, ----, S_i\}.$$

The information provided by individual symbol is known as self information which is denoted by $I(S_i)$ and is given by,

Note: Information rate: If the time rate at which source emits 'r' symbol then information rate is $R = r.H(S)$

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$$I(S_i) = \log_2 \frac{1}{P(S_i)} \quad \text{(bits)}$$

Where $P(S_i)$ is the probability of occurence of each symbol.

Few facts about information is as follows:

i) $I(S_k) > I(S_i)$ if $P(S_k) < P(S_i)$

ii) $I(S_k) < I(S_i)$ if $P(S_k) > P(S_i)$

iii) $I(S_k) \rightarrow 1$ if $P(S_k) \rightarrow 0$

iv) $I(S_k) \rightarrow 0$ if $P(S_k) \rightarrow 1$

The average information (entropy) of sequence of symbol is the average information of source itself and is denoted by H which is given by,

$$H(S) = \sum_{i=1} P(S_i) \log_2 \frac{1}{P(S_i)} \quad \text{(bits)}$$

Q.1. Consider a binary source $S = \{0, 1\}$ with probabilities $P(0) = 0.5$, $P(1) = 0.5$, then calculate its self information.

$$I(0) = \log_2 \frac{1}{P(S_0)} = \log_2 \frac{1}{0.5}$$
$$= 1$$

$$I(1) = \log_2 \frac{1}{P(S_1)} = \log_2 \frac{1}{0.5}$$
$$= 1$$

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* Shannon-Fano Coding:

(decreasing order probabi..)

Q. The source of information A generates the symbols {A_0, A_1, A_2, A_3, A_4} with corresponding probabilities {0.4, 0.3, 0.15, 0.1, 0.05}. Encode the source symbols using (i) binary encoder
(ii) Shannon-Fano encoder.
(iii) Binary Huffman Coding & Compare their efficiency.

⇒ soln

The required table is shown below:

Source Symbol	P_i	Binary code	Shannon-Fano Code	Binary Huffman Code
A_0	0.4	000	0	1
A_1	0.3	001	10	00
A_2	0.15	010	110	010
A_3	0.1	011	1110	0110
A_4	0.05	100	1111	0111

Since, we have 5 symbols, we need 3 bits at least to represent each symbol in binary code.

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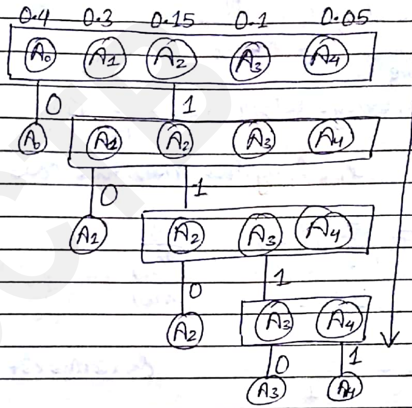
Shannon-Fano Code:



make equal half or close to half

left → 0
right → 1

Binary Huffman coding

S			S_1		S_2		S_3	
A_0	0.4	1	0.4	1	0.4	1	0.6	0
A_1	0.3	00	0.3	00	0.3	00	0.4	1
A_2	0.15	010	0.15	010	0.3	01		
A_3	0.1	0110	0.15	011				
A_4	0.05	0111						

Hence, Binary Huffman Code for 5 symbols are:

Symbol	Code
A_0	1
A_1	00
A_2	010
A_3	0110
A_4	0111

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Note:

$$\text{Efficiency}, \eta = \frac{H}{L_{avg}}$$

where

$$\text{Average length}, L_{avg} = \sum_{i=1}^{n} P_i l_i$$

l_i = length of individual code

$$H(S) = \sum_{i=1}^{5} P(S_i) \log_2 \frac{1}{P(S_i)}$$

$= 2.008$ bits/symbol

$$(L_{avg})_{BL} = \sum_{i=1}^{5} P_i l_i \quad 0.4 \times 3 + 0.3 \times 3 + \dots$$

$= 3$ bits/symbol

$(L_{avg})_{SF} = 2.05$ bits/symbol $\left(0.4 \times 1 + 0.3 \times 2 + 0.15 \times 3 + 0.1 \times 4 + 0.05 \times 4\right)$

$(L_{avg})_{BHL} = 2.05$ bits/symbol

$$\eta_{BL} = \frac{2.008}{3} = 0.669 = 66.9\%$$

$$\eta_{SF} = \frac{2.008}{2.05} = 0.979 = 97.9\%$$

$$\eta_{BHL} = \frac{2.008}{2.05} = 0.979 = 97.9\%$$

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* Hamming Distance, Hamming Weight, Minimum

Hamming Distance:

Hamming Distance between two code words or code vectors is the no. of differences between corresponding bits.

eg: $d(0000, 1111) = 4$

$d(0000, 1001) = 2$

Hamming Weight is the no. of non-zeros in a code vectors. eg: Hamming weight of code vector 0101 is 2 & 1111 is 4.

The minimum distance or minimum hamming distance is the smallest hamming distance between all possible pairs in a set of code vectors. Minimum hamming distance of a following pair is: $d(0000, 0001, 0011, 1111)$

$d_{min} = 1$

Now to guarentee the detection upto 's' errors, the minimum hamming distance is $d_{min} = s+1$. and to guarentee the correction upto 't' errors, the minimum hamming distance is:

$d_{min} = 2t + 1$.

n,k
input $k = 0000$
output $n = 0000111$
parity $q = 111$

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Q. Suppose a code has a hamming distance $d_{min} = 4$, what is the error detection and correction capacity of this scheme.

⇒ error detection

$$d_{min} = s+1$$
$$s = d_{min} - 1$$
$$= 4-1$$
$$= 3$$

Hence, a code detect 3 errors,

Correction:

$$d_{min} = 2t+1$$
$$4 = 2t+1$$
$$t = \frac{4-1}{2} = 3/2 = 1.5$$
$$= 2$$

Hence, correction capacity of scheme is 2.

Error Control Coding:

Input message bits (k-bits) → Channel Encoder → Code block output (n-bits)

During transmission process, the transmitted signal passes through some noisy channel causing error in the received data. These errors can be detected and corrected using coding technique. Coding techniques add some extra bits to the message bits and these bits are used to detect errors at the receiver. Although these extra bits reduce the bit rate of transmitter, the advantage is that error probability is reduced.

mode 2 ⟺ X-OR same - 0
diff - 1

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Linear Block Coding:

Input message → Channel Encoder → Code Block output

Message
← k bits →

Message	Check Bits
← k bits →	← $q = n-k$ bits →
← n bits →	

For the block of k-message bits, $q = n-k$ parity bits or check bits are added. This means that the total bits at the output of channel encoder is 'n'. Such type of code are known as (n,k) block code.

A code is known as linear, if the sum of any two code vectors produces another code vector. Here, sum is performed according to mode 2 adding rule. In general linear block codes are described in a matrix form, The steps for determining code words for linear block code is as follows:

1. Let the particular code vector consist of $m_1, m_2, \ldots, m_k$ message bits and check bits be $C_1, C_2, \ldots, C_q$ then code vector may be written as $X = (m_1, m_2, \ldots, m_k, C_1, C_2, \ldots, C_q)$

2. Here the check-bits play the role of error detection and correction and the function of linear block code is to generate these check bits. The code vector is represented in terms of generator-

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matrices;

$$[X]_{1\times n} = [M]_{1\times k} \times [G]_{k\times n}$$

Where, G = Generator matrix $= [I_k | P_{k\times q}]_{k\times n}$

I_k = identity matrix.

$$P_{k\times q} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ \vdots & \vdots & & \vdots \\ P_{k1} & P_{k2} & \cdots & P_{kq} \end{bmatrix}_{k\times q}$$

3. Now check vector or parity vector may be obtained as: $C = MP$

$$[C_1\ C_2 \cdots C_q] = [m_1\ m_2 \cdots m_k] \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ \vdots & \vdots & \cdots & \vdots \\ P_{k1} & P_{k2} & \cdots & P_{kq} \end{bmatrix}$$

Q. The generator matrix for 2(6,3) block code is shown below. Obtain all code words of this code.

$$G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

⇒ Here,
$n = 6$
$k = 3$
$\therefore q = n - k = 6 - 3 = 3$

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So,
from G Matrix,

$$P_{k\times q} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Here,
the message vector is of 3-bits, and so there'll be a total of $2^k = 2^3 = 8$ possible message combination.
Now, Check bits can be calculated as follows,

$$C = M \cdot P$$

$$[C_1\ C_2\ C_3] = [m_1\ m_2\ m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

XOR

$\therefore C_1 = m_2 \oplus m_3$
$C_2 = m_1 \oplus m_3$
$C_3 = m_1 \oplus m_2$

Now, the complete code vector can be calculated using following table.

message bits			Check bits			Complete Code vector
m_1	m_2	m_3	C_1 $(m_2 \oplus m_3)$	C_2 $(m_1 \oplus m_3)$	C_3 $(m_1 \oplus m_2)$	m.c (format write)
0	0	0	0	0	0	000 000
0	0	1	1	1	0	001 110
0	1	0	1	0	1	010 101
0	1	1	0	1	1	011 011
1	0	0	0	1	1	100 011
1	0	1	1	0	1	101 101
1	1	0	1	1	0	110 110
1	1	1	0	0	0	111 000

* Binary Cyclic Code (BCC):

Cyclic codes may be described as the subclass of linear block code. They have the property that a cyclic shift of one code word produces another code word. As an example, let us consider an n-bit code vector as,

$$X = (x_{n-1}, x_{n-2}, \ldots, x_0)$$

Where, $x_{n-1}, x_{n-2}, \ldots, x_0$ represents bits of code vector. A linear code is known as cyclic code if every cyclic shift of the code vector produces another code vector.

If X is shifted by 1-bit then,

$$X' = (x_{n-2}, x_{n-1}, \ldots, x_0, x_{-1}),$$

is also a code vector.